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The Leontief Paradox, Reconsidered

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Using the Heckscher-Ohlin-Vanek model of trade, it is shown that a country is revealed to be relatively well endowed in capital compared with labor if and only if one of the following three conditions holds, where K_x, K_m, L_x, L_m, K_c, L_c are capital and labor embodied in exports, imports, and consumption: (*a*) $K_x - K_m > 0, L_x - L_m < 0$; (*b*) $K_x - K_m > 0, L_x - L_m > 0, (K_x - K_m)/(L_x - L_m) > K_c/L_c$; (*c*) $K_x - K_m < 0, L_x - L_m < 0, (K_x - K_m)/(L_x - L_m) < K_c/L_c$. Leontief's data for the United States in 1947 satisfy *b*, and the United States is actually revealed by trade to be capital abundant. The comparison by Leontief of K_x/L_x with K_m/L_m is shown to be theoretically inappropriate.

The Leontief paradox (1954) rests on a simple conceptual misunderstanding. It makes use of the intuitively appealing but nonetheless false proposition that if the capital per man embodied in exports is less than the capital per man embodied in imports, the country is revealed to be poorly endowed in capital relative to labor. This is a true proposition if the net export of labor services is of the opposite sign of the net export of capital services, but when both are positive, as in Leontief's data, the proper comparison is between the capital per man embodied in *net* exports and the capital per man embodied in consumption. Leontief's figures, which produced the so-called paradoxical result that U.S. exports are less capital intensive than U.S. competing imports, can also be used to show that U.S. net exports are more capital intensive than U.S. consumption, which in fact implies

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that capital is abundant relative to labor. There is no paradox if the conceptually correct calculations are made.

The first section of this paper shows that a country is revealed to be relatively well endowed in capital compared with labor if and only if one of the following three conditions holds, where $K_x, K_m, L_x, L_m, K_c, L_c$ are capital and labor embodied in exports, imports, and consumption:

$$a) \quad K_x - K_m > 0,\ L_x - L_m < 0.$$

$$b) \quad K_x - K_m > 0,\ L_x - L_m > 0,\ (K_x - K_m)/(L_x - L_m) > K_c/L_c.$$

$$c) \quad K_x - K_m < 0,\ L_x - L_m < 0,\ (K_x - K_m)/(L_x - L_m) < K_c/L_c.$$

Although Leontief found that $K_x/L_x < K_m/L_m$, his data are shown in Section II to satisfy b, and therefore the United States is revealed to be capital abundant. In a largely overlooked article, Williams (1970) makes a related point.

I. Trade-revealed Factor Abundance

This reconsideration of the Leontief paradox rests on the Heckscher-Ohlin-Vanek (HOV) theorem (Vanek 1968).

The Heckscher-Ohlin-Vanek Theorem

Given: (*a*) There are n commodities which are freely mobile internationally. (*b*) There are n factors which are perfectly immobile internationally. (*c*) All individuals have identical homothetic preferences. (*d*) Production functions are the same in all countries and exhibit constant returns to scale. (*e*) There is perfect competition in the goods and factors markets. (*f*) Factor prices are equalized across countries.

Then: There exists a set of positive scalars $\alpha_i, i = 1, \ldots, I$, such that the vector of net exports of country i, T_i, the vector of factor endowments of country i, E_i, and the $n \times n$ matrix of total factor requirements A, bear the following relationship to each other:

$$AT_i = E_i - E_w\alpha_i, \qquad i = 1, \ldots, I, \tag{1}$$

where E_w is the world's endowment vector, $E_w = \sum_i E_i$.

Proof: The proof of this result is straightforward. The equalization of factor prices and constant-returns-to-scale production functions imply the matrix of total factor inputs A, where A_{jk} is the amount of factor j used to produce one unit of commodity k. If Q_i is the vector of outputs of country i, then equilibrium in the factor markets requires

factor demand equal to factor supply $AQ_i = E_i$. The summation of this equation over all countries produces $AQ_w = E_w$. Then, identical homothetic tastes imply that the consumption vectors C_i of each country are proportional to each other and also proportional to world output Q_w: $C_i = Q_w\alpha_i$. Country i's trade is $T_i = Q_i - C_i$, and the factors embodied in trade are $AT_i = A(Q_i - C_i) = E_i - AQ_w\alpha_i = E_i - E_w\alpha_i$.

The set of equations (1) serves as a logically sound foundation for a study of trade-revealed factor abundance. Two of these equations describe the relationship between capital and labor endowments and the implicit trade in capital and labor services:

$$K_T = K_i - \alpha_i K_w, \tag{2a}$$

$$L_T = L_i - \alpha_i L_w, \tag{2b}$$

where (K_T, L_T) are capital and labor embodied in net exports, (K_i, L_i) are the factor endowments of country i, and (K_w, L_w) are the world's factor endowments.

We take the following definition of factor abundance.

Definition: Capital in country i is said to be abundant in comparison with labor if and only if the share of the world's capital stock located in i exceeds the share of the world's labor force: $K_i/K_w > L_i/L_w$.

Factor abundance is revealed by trade through a comparison of the vector of factors used to produce various vectors of commodities. These vectors may be defined as follows.

Definition: The vector of factors embodied in the vector of commodities z is Az, where A is the matrix of total factor requirements.

The following result establishes necessary and sufficient conditions for trade to reveal an abundance of capital.

Corollary 1
Capital is revealed by trade to be abundant relative to labor if and only if

$$K_i/(K_i - K_T) > L_i/(L_i - L_T). \tag{3}$$

Proof: Equations (2a) and (2b) can be rewritten as

$$K_w = (K_i - K_T)/\alpha_i,$$

$$L_w = (L_i - L_T)/\alpha_i.$$

Thus

$$K_i/K_w = \alpha_i K_i/(K_i - K_T),$$

$$L_i/L_w = \alpha_i L_i/(L_i - L_T),$$

from which (3) is a consequence.

There are three useful ways of rewriting (3). If K_c is the amount of capital embodied in the commodities used in country i, then $K_i - K_T = K_c$ and, similarly, $L_i - L_T = L_c$. Then (3) is equivalent to

$$K_i/L_i > K_c/L_c, \tag{3a}$$

which means that a country is revealed to be capital abundant if its production is more capital intensive than its consumption.

Another way to rewrite (3) is $K_i(L_i - L_T) > L_i(K_i - K_T)$, or

$$-K_iL_T > -L_iK_T. \tag{3b}$$

If L_T is positive, then this inequality becomes $K_T/L_T > K_i/L_i$, or $K_T/K_i > L_T/L_i$. Thus a country which is an exporter of both labor services and capital services is revealed by trade to be relatively capital abundant if trade is more capital intensive than production or, equivalently, if the share of capital exported exceeds the share of labor exported. Similarly, if L_T is negative the inequalities are reversed, and a country which is an importer of both labor services and capital services is revealed by trade to be relatively capital abundant if trade is less capital intensive than production or, equivalently, if the share of capital imported is less than the share of labor imported.

Yet another possibility is to rewrite (3b) as $-(K_c + K_T)L_T > -(L_c + L_T)K_T$, or

$$-K_cL_T > -L_cK_T. \tag{3c}$$

Thus a country which is an exporter of both labor services and capital services is revealed by trade to be relatively capital abundant if the capital intensity of net exports exceeds the capital intensity of consumption, $K_T/L_T > K_c/L_c$, and a country which is an importer of both capital and labor services is revealed by trade to be capital abundant if the capital intensity of net exports is less than the capital intensity of consumption, $K_T/L_T < K_c/L_c$.[1]

Inequalities (3a), (3b), and (3c) identify three equivalent ways of computing trade-revealed factor abundance. Trade even more directly reveals relative capital abundance if the services of one factor are exported and the services of the other are imported, since in-

[1] It may be observed that Williams (1970) uses (2) to form his equation (23): $(K_w - K_i)/K_i = (1/\alpha_i) - [(K_T + \alpha_iK_i)/\alpha_iK_i]$, which he calls the "plentifulness ratio." This formula suggests erroneously that the consumption share α_i is necessary to infer the relative abundance of capital. Moreover, Williams (1970, p. 121) reports that "the percentage of United States net capital, labour and natural resources exported as 7.14, 4.24, and 3.55, respectively. Intuition would suggest that, under these circumstances the United States must be implicitly plentiful in capital." Actually, this is enough (see his eq. 36) to establish the capital abundance of the United States, given $K_T > 0$, $L_T > 0$. This is discussed further below.

equality (3b) is satisfied if $K_T > 0$ and $L_T < 0$ and is violated if $K_T < 0$ and $L_T > 0$. For reference, this will be stated as a corollary.

Corollary 2
If the net export of capital services and the net export of labor services are opposite in sign, then the factor with positive net exports is revealed to be the relatively abundant factor.

Corollaries 1 and 2 imply that one should be examining the factor content of *net* exports, but the tradition beginning with Leontief is to distinguish exports from imports. In some cases, this is an equivalent procedure.

Corollary 3
Given that the net export of capital services and the net export of labor services are opposite in sign, then the capital per man embodied in exports (K_x/L_x) exceeds the capital per man embodied in imports (K_m/L_m) if and only if the country is relatively abundant in capital, $K_i/K_w > L_i/L_w$.

Proof: Suppose first that $K_T > 0$ and $L_T < 0$; then by corollary 2, $K_i/K_w > L_i/L_w$. But $0 < K_T = K_x - K_m$ implies $K_x/K_m > 1$, and $0 > L_T = L_x - L_m$ implies $1 > L_x/L_m$. Thus $K_x/K_m > L_x/L_m$, and $K_x/L_x > K_m/L_m$. Similarly $K_T < 0$ and $L_T > 0$ imply both $K_i/K_w < L_i/L_w$ and $K_x/L_x < K_m/L_m$.

A substantial practical defect of corollary 3 is that it assumes that K_T and L_T are opposite in sign. In fact, using Leontief's 1947 data, K_T and L_T are both positive: The United States exported both capital services and labor services. In that event, the ordering $K_x/L_x < K_m/L_m$ reveals nothing about the relative magnitudes of K_i/K_w and L_i/L_w.

Corollary 4
If there are more than two commodities, the ordering of exports and imports by factor intensity, say $K_x/L_x > K_m/L_m$, is compatible with either order of factor abundance, $K_i/K_w < L_i/L_w$ or $K_i/K_w > L_i/L_w$.

Proof: An example of the "paradoxical" case $K_x/L_x < K_m/L_m$ and $K_i/K_w > L_i/L_w$ will suffice. Let the factor requirements matrix be given as

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 3 & 2 & .5 \\ 1 & 0 & 3 \end{bmatrix}$$

where the first row corresponds to capital inputs, the second row to labor inputs, and the third to land inputs. Suppose that the output vectors are given by

$$Q_i = (8, 16, 5)'$$

and

$$Q_w = (12, 68, 52)'.$$

The endowment vectors are then

$$AQ_i = E_i = (53, 58.5, 23)'$$

and

$$AQ_w = E_w = (168, 198, 168)'.$$

If the prices of the commodities are all one, then trade balance, $0 = 1'T_i$, implies

$$\alpha_i = \frac{1'Q_i}{1'Q_w} = \frac{29}{132} = .22.$$

Using this, and the endowment vectors, we can compute the excess factor supplies

$$(E_i - \alpha_i E_w)' = (53, 58.5, 23) - .22(168, 198, 168)$$

$$= (16.04, 14.94, -13.96).$$

Therefore, country i, on net, exports the services of both capital and labor and imports the services of land. The commodity trade vector implied by the above system is

$$T_i = (5.36, 1.04, -6.44)'.$$

Partitioning this into two vectors, exports (X_i) and imports (M_i), we obtain

$$X_i = (5.36, 1.04, 0)'$$

and

$$M_i = (0, 0, 6.44)'.$$

Computing the factor content of exports and imports separately we have

$$AX_i = (22.48, 18.16, 5.36)'$$

and

$$AM_i = (6.44, 3.22, 19.32)'.$$

Thus, for example, country i exports 22.64 units of capital and imports 6.44 units. Computing the capital-labor content ratio we obtain

$$\lambda = \frac{(K_x/L_x)}{(K_m/L_m)} = \frac{1.24}{2} = .62,$$

which is less than one. From this we might, as does Leontief, erroneously conclude that capital is scarce relative to labor in this country. However, the true ordering of factor abundance is given by the ratio of country i's endowment to the world's endowment. Computing these ratios for each factor we obtain

$$\frac{K_i}{K_w} = .315,$$

$$\frac{L_i}{L_w} = .295.$$

This ranking indicates that contrary to the inference based on λ, the country is *abundant* in capital relative to labor.

Corollary 4 indicates that Leontief's method of computing trade-revealed factor abundance orderings is erroneous. However, in the unlikely world of two commodities, it is a correct method.

Corollary 5
If there are only two commodities, and if one is exported and the other is imported, the ordering of exports and imports by capital intensity is the same as the ordering of factor abundance; that is, $K_x/L_x \geq K_m/L_m$ if and only if $K_i/K_w \geq L_i/L_w$.

Proof: It is necessary to show that a capital-abundant country exports the capital-intensive good, assuming one good is exported and the other is imported. If X and M are the quantity of exports and imports, then equation (1) can be written as

$$A_{Kx}X - A_{Km}M = K_i - \alpha_i K_w,$$

$$A_{Lx}X - A_{Lm}M = L_i - \alpha_i L_w.$$

The ordering $K_i/K_w \geq L_i/L_w$ is equivalent to $(A_{Kx}X - A_{Km}M)/K_w \geq (A_{Lx}X - A_{Lm}M)/L_w$, which can be rewritten as

$$X\left(\frac{A_{Kx}}{A_{Lx}} - \frac{K_w}{L_w}\right) \geq M\left(\frac{A_{Km}}{A_{Lm}} - \frac{K_w}{L_w}\right)\left(\frac{A_{Lm}}{A_{Lx}}\right).$$

The world's capital-labor ratio K_w/L_w must be between the industry intensity ratios A_{Kx}/A_{Lx} and A_{Km}/A_{Lm}, which implies that the left or

right sides of the inequality above are opposite in sign, which is compatible only with $A_{Kx}/A_{Lx} > A_{Km}/A_{Lm}$. Thus $K_i/K_w \geqslant L_i/L_w$ is equivalent to $A_{Kx}/A_{Lx} > A_{Km}/A_{Lm}$.

II. Leontief's Data Reexamined

Tables 1, 2, and 3 contain information extracted from Leontief (1954) and from Travis (1964). Table 1 is Leontief's basic summary table, which reveals that $K_x/L_x < K_m/L_m$. But table 2 indicates that the United States in 1947 was a net exporter of both capital services and labor services. For this reason, the information contained in table 1 does not reveal the relative factor abundance of capital and labor (see corollary 4). The appropriate comparison, as described in corollary 1, is reported in table 3. Since net exports are much more capital intensive than consumption, the United States is revealed by its trade to be relatively well endowed in capital compared with labor.[2]

Finally, it is necessary to comment on why the United States had such a large trade surplus according to the data in table 2. This is partly due to the fact that "noncompeting" imports, such as coffee, tea, and jute, have been eliminated from the vector of imports. It is difficult to find a theoretically sound justification for this procedure. The HOV theorem uses the factor-price-equalization theorem, which requires incomplete specialization. It is necessary, therefore, to imagine that the United States in fact produces at least small amounts of coffee, tea, and jute, and so forth. It is natural to suppose that the

TABLE 1

DOMESTIC CAPITAL AND LABOR REQUIREMENTS PER MILLION DOLLARS OF UNITED STATES EXPORTS AND OF COMPETITIVENESS IMPORT REPLACEMENTS (of Average 1947 Composition)

	Exports	Imports
Capital ($, 1947 prices)	2,550,780	3,091,339
Labor (man-years)	182.313	170.004

SOURCE.—Leontief (1954, sec. VI).

[2] Baldwin's (1971) finding that the Leontief paradox holds also for 1962 data cannot be explained away so easily. Baldwin reports capital in 1958 dollars embodied in a million (1958) dollars of imports and exports to be $2,132,000 and $1,876,000, respectively. The corresponding man-year figures are 119 and 131. Merchandise exports in millions of 1962 dollars were 20,781 and merchandise imports were 16,260. As in 1947 the United States was a net exporter of both capital services and labor services, $K_T > 0, L_T > 0$, but the ratio had fallen to $K_T/L_T = \$5,579$ in 1958 dollars per man year. This number falls below Travis's estimate of the 1947 capital per man equal to $6,949/man year and is likely to fall below any estimates for 1962 as well.

TABLE 2

ADDITIONAL INFORMATION ON TRADE AND ENDOWMENTS

Trade or Factor	Value
Exports	$16,678.4 million
Imports (competitive)	$ 6,175.7 million
Net exports of capital services (K_T)	$23,450 million
Net exports of labor services (L_T)	1.990 million man-years
Capital-labor intensity of trade (K_T/L_T)	$11,783 /man-year

SOURCE.—Leontief (1954, table 2, n.).

TABLE 3

CAPITAL INTENSITY OF CONSUMPTION, PRODUCTION, AND TRADE

	Production	Net Exports	Consumption*
Capital	$328.519 million	$23,450 million	$305,069 million
Labor	47.273 million man-years	1.99 million man-years	45.28 million man-years
Capital/labor	$6,949/man-year	$11,783/man-year	$6,737/man-year

SOURCE.—For production figures, Travis (1964).
*Uses the identity, Consumption = Production − Net Exports.

production of these commodities uses capital, labor, and "tropical land" which is very scarce in the United States. But any capital and labor embodied in the imports of "noncompeting" goods should be included in the above calculations. May we suppose that these products are labor intensive, which works also to explain the Leontief paradox?

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