Problem Set 3

UCSD
ECON 245
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Note: Due Data is March 5.

Problem 0. Eaton and Kortum and the Fréchet Math. Consider the basic Eaton and Kortum model seen in class where the price for a variety ω from origin i in destination j is given by:

$$p_{ij}(\omega) = \frac{w_i}{z_i(\omega)}\tau_{ij}.$$

In each country draws the productivity for each variety $\omega \in [0,1]$ from a country-specific Fréchet distribution:

$$F_i(z) = \exp\left(-T_i z^{-\theta}\right) \quad T_i > 0 \quad \theta > 1$$

Derive the expression for the fraction of goods that country j buys from country i, i.e., the fraction of goods for which $p_{ij}(\omega) = \min_k\{p_{kj}\}$. Denote this fraction by π_{ij}.
Bonus: Derive the distribution of prices among the goods j imports from j, that is compute the CDF of prices for goods j buys from i. Hint: this should come out to be independent of the origin country i.

Problem 1. The Basic Armington Model with Free Labor Mobility. Consider the exact same Armington model as in the last problem set. Now we introduce one difference: workers can move freely. This introduces an additional equilibrium condition: the real wage $W_i = U_i w_i / P_i$ has to be equal across locations in equilibrium. Solve for the equilibrium with $\tau_{ij} = 2$ and for the one with $\tau_{ij} = 1$ separately. Graph the change in local population between the two scenarios against location productivity. The local amenities, U_i are mirror images of the productivity parameter A_i. The region with $A_i = 1$ has $U_i = 10$, the one with $A_i = 2$ has $U_i = 9$ and so on. Hint: relative to the old code you should now add an "outer loop." First solve for the wage on the inner loop holding population constant. Then compute the real wage in each location. In the outer loop, you then add some population to locations which have a real wage higher than the median region, and take workers away from regions with a real wage below the median. In this way you iterate between updating wages holding populations fixed, and updating populations holding wages fixed.

Problem 2. The Basic Armington Model with Frictional Labor Mobility. Same problem as Problem 1 above but now workers obtain an idiosyncratic preference shock ξ_i for each location which is drawn from a Frechet distribution, $F(\xi) = \exp(-\xi^{-\theta})$. Choose $\theta = 8$. Solve for the equilibrium with $\tau_{ij} = 2$ and for the one with $\tau_{ij} = 1$ separately. Graph the change in local population between the two scenarios against location productivity. Hint: relative to the old code you should now add an "outer loop." First solve for the wage on the inner loop holding population constant. Then compute the real wage in each location. The outer loop is now simpler: you just need to compute the population distribution implied by the wages. The Frechet assumption gives you an analytical expression for the fraction of workers in each location as a function of the spatial distribution of wages.