Problem Set 2

UCSD
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Instructor: Fabian Eckert

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Note: This problem set is developing. We will continue to extend the model and our code. And eventually take it to the data. I will post an updated version of this problem set each week or so. For now Task 1 is due February February 12.

Problem 0. Constant Elasticity Demand. Consider a consumer with the following preferences over a set Ω of differentiated varieties:

$$U_j = \left(\int_{\omega\in\Omega} a_{ij}(\omega)^{1/\sigma} c_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}}$$
$$= \left(\sum_{i\in S}\int_{\omega\in\Omega_i} a_{ij}(\omega)^{1/\sigma} c_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}},$$

where Ω_i is the mass of varieties produced in location i, $c_{ij}(\omega)$ is consumption of variety ω from origin country i in location j. who faces the following budget constraint:

$$\sum_{i\in S}\int_{\omega\in\Omega_i} p_{ij}(\omega)c_{ij}(\omega) \leq X_j$$

where X_j is total expenditure of the representative consumer in location j. i and j index locations. Solve the consumer's problem and derive and expression for their demand for variety ω from location j. Show step by step derivations. *Hint:* Use the following shorthand for the "optimal price index:"

$$P_j = \left(\sum_{i\in S}\int_{\omega\in\Omega_i} a_{ij}(\omega)p_{ij}^{1-\sigma}(\omega)d\omega\right)^{\frac{1}{1-\sigma}}$$

Solution P0. Set up the problem:

$$\max_{\{q_{ij}(\omega)\}_{\omega\in\Omega}} \left(\sum_{i\in\Omega} a_{ij}(\omega)^{\frac{1}{\sigma}} q_{ij}(\omega)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \text{ s.t. } \sum_{\omega\in\Omega} q_{ij}(\omega)p_{ij}(\omega) \leq X_j$$

There is an implicit constraint on the choice variable: $q_{ij}(\omega) > 0$. We can ignore it since CES preferences imply marginal utility rises to infinity as quantity consumed goes to zero. We now set up the Lagrangian:

$$\mathcal{L} : \left(\sum_{\omega\in\Omega} a_{ij}(\omega)^{\frac{1}{\sigma}} q_{ij}(\omega)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - \lambda\left(\sum_{\omega\in\Omega} q_{ij}(\omega)p_{ij}(\omega) - X_j\right)$$

First order conditions wrt to the optimal quantity imply:

$$\left(\sum_{\omega\in\Omega} a_{ij}(\omega)^{\frac{1}{\sigma}} q_{ij}(\omega)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} a_{ij}(\omega)^{\frac{1}{\sigma}} q_{ij}(\omega)^{-\frac{1}{\sigma}} = \lambda p_{ij}(\omega)$$

As always, the FOC wrt to Lagrange multiplier is just the budget constraint. Dividing the FOCs for two different varieties:

$$\frac{a_{ij}(\omega)}{a_{ij}(\omega')} = \frac{p_{ij}^{\sigma}(\omega)}{p_{ij}^{\sigma}(\omega')}\frac{g_{ij}(\omega)}{q_{ij}(\omega')}$$

Rearrange and multiply both sides by $p_{ij}(\omega)$:

$$q_{ij}(\omega')p_{ij}(\omega') = \frac{1}{a_{ij}(\omega)}q_{ij}(\omega)p_{ij}(\omega)^{\sigma}a_{ij}(\omega')p_{ij}^{1-\sigma}(\omega)$$

Now we simply some over $\omega' \in \Omega$ to obtain:

$$\sum_{\omega' \in \Omega} q_{ij}(\omega')p_{ij}(\omega') = \frac{1}{a_{ij}(\omega)}q_{ij}(\omega)p_{ij}(\omega)^{\sigma}\sum_{\omega' \in \Omega} a_{ij}(\omega')p_{ij}^{1-\sigma}(\omega)$$

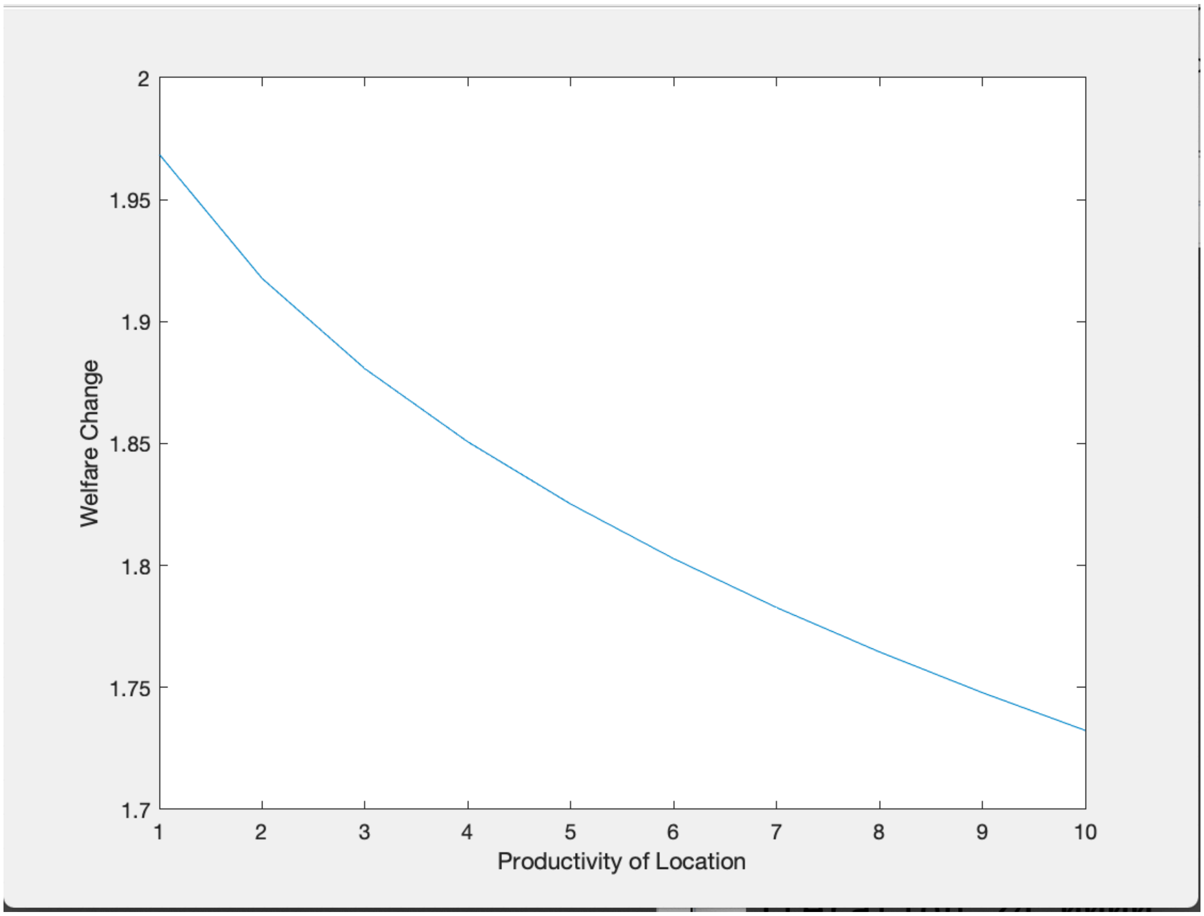
Using the hint we can write this as:

$$X_j = \frac{1}{a_{ij}(\omega)}q_{ij}(\omega)p_{ij}(\omega)^{\sigma}P_j^{1-\sigma}$$

which is our CES demand we have used throughout the last few lectures!

Problem 1. The Basic Armington Model without Labor Mobility. Consider the following model setup. There is a discrete set S of locations. Each location produces its own distinct variety of a differentiated good. Within each location there is perfect competition among firms. All firms produce with the same labor-only technology: $y = A_i l_i$, where A_i is a location-specific productivity term. Workers in each location inelastically provide one unit of labor to the local labor market and spend on all their income on consuming the differentiated varieties of the good. They have CES preferences as in Problem 0 above. There are iceberg trade costs $\tau_{ij} \geq 0$ between locations, so that τ_{ij} have to be shipped from i of a given good for 1 unit to arrive in destination j.
(a.) Assume the following parameters: $S = 10$, $a_{ij} = 1 \forall i, j \in S$, $\sigma = 2$. Also assume $L_i = 1 \forall i$ $A_i = 1, 2, 3, ..., 10$. Solve for the equilibrium of this model when $\tau_{ii} = 1, \tau_{ij} = 2 \forall i \neq j$. Then re-compute the equilibrium when $\tau_{ii} = 1, \tau_{ij} = 1 \forall i \neq j$. Graph the welfare gains from the trade cost reduction in each region against its underlying productivity. Verify that the formula for welfare gains from trade cost changes we derived in class holds in your model.

Solution P1. Find attached the code to solve this problem. The figure shows that welfare is rising most in the least productive locations. This reflects that these locations benefit most since they get access to very productive world markets. The more productive locations get access to countries less productive than themselves and hence benefit less, since they end up buying a larger share of products from their own producers.


2
1.95
1.9
1.85
1.8
1.75
1.7
Welfare Change
1
2
3
4
5
6
7
8
9
10
Productivity of Location