Problem Set 2

UCSD
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Note: This problem set is developing. We will continue to extend the model and our code. And eventually take it to the data. I will post an updated version of this problem set each week or so. For now Task 1 is due February February 12.

Problem 0. Constant Elasticity Demand. Consider a consumer with the following preferences over a set Ω of differentiated varieties:

$$U_j = \left(\int_{\omega \in \Omega} a_{ij}(\omega)^{1/\sigma} c_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$
$$= \left(\sum_{i \in S} \int_{\omega \in \Omega_i} a_{ij}(\omega)^{1/\sigma} c_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where Ω_i is the mass of varieties produced in location i, $c_{ij}(\omega)$ is consumption of variety ω from origin country i in location j. who faces the following budget constraint:

$$\sum_{i \in S} \int_{\omega \in \Omega_i} p_{ij}(\omega) c_{ij}(\omega) \leq X_j$$

where X_j is total expenditure of the representative consumer in location j. i and j index locations. Solve the consumer's problem and derive and expression for their demand for variety ω from location j. Show step by step derivations. *Hint:* Use the following shorthand for the "optimal price index:"

$$P_j = \left(\sum_{i \in S} \int_{\omega \in \Omega_i} a_{ij}(\omega) p_{ij}^{1-\sigma}(\omega) d\omega \right)^{\frac{1}{1-\sigma}}$$

Problem 1. The Basic Armington Model without Labor Mobility. Consider the following model setup. There is a discrete set S of locations. Each location produces its own distinct variety of a differentiated good. Within each location there is perfect competition among firms. All firms produce with the same labor-only technology: $y = A_i l_i$, where A_i is a location-specific productivity term. Workers in each location inelastically provide one unit of labor to the local labor market and spend on all their income on consuming the differentiated varieties of the good. They have CES preferences as in Problem 0 above. There are iceberg trade costs $\tau_{ij} \geq 0$ between locations, so that τ_{ij} have to be shipped from i of a given good for 1 unit to arrive in destination j.

(a.) Assume the following parameters: $S = 10$, $a_{ij} = 1 \forall i, j \in S$, $\sigma = 2$. Also assume $L_i = 1 \forall i$ $A_i = 1, 2, 3, ..., 10$. Solve for the equilibrium of this model when $\tau_{ii} = 1, \tau_{ij} = 2 \forall i \neq j$. Then re-compute the equilibrium when $\tau_{ii} = 1, \tau_{ij} = 1 \forall i \neq j$. Graph the welfare gains from the trade cost reduction in each region against its underlying productivity. Verify that the formula for welfare gains from trade cost changes we derived in class holds in your model.