INTERNATIONAL TRADE - ECON 245

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SPATIAL EQUILIBRIUM

INTRODUCTION

- The workhorse spatial economics framework is based on the papers of Rosen (1979) and Roback (1982), simply known as "Rosen-Roback"
  - Many many applications and tests over the years
- Central insight:
  - Labor mobility across location leads to a spatial equilibrium *whereby high wages are offset by high prices, and higher real wages by negative amenities, i.e., such that overall utility equalizes across space*

STATIC LOCATION DECISIONS

ROSEN-ROBACK

SETUP

- There is a set S of discrete locations indexed by i
- Market structure is perfect competition
- All locations produce a single homogeneous good that is freely traded
- Workers can move freely across space
- Locations differ in:
  - Amenities, U_i
  - Productivities, A_i

PRODUCTION

- All firms produce a homogeneous good
- Firms in each location produce with constant returns to scale
- Technology is labor-only and productivity denoted A_i: $y_i = A_i L_i$
- We allow for the possibilities of production externalities:

$$A_i = \bar{A}_i L_i^{\alpha} \quad \text{where} \quad \alpha \geq 0$$

where $\bar{A}_i$ is the fundamental part of productivity

- With perfect competition the price is equal to marginal product: $p = w_i / A_i$

CONSUMERS

- Consumers spend all their money on the homogeneous good
- The consumer also enjoys the location specific amenity U_i so that their total utility in location i is given by

$$W_i = U_i \frac{w_i}{p_i}$$

- We also allow for amenity spillovers:

$$U_i = \bar{U}_i L_i^{-\beta} \quad \text{where} \quad \beta \geq 0$$

- Consumers do not internalize their effect on productivties/amenities

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- The good is homogeneous and freely traded so its price has to be the same everywhere
  - We choose its price as the numeraire $p = 1$
    - This implies that $w_i = A_i$
- Plugging this into the utility function we solve for the indirect utility:

$$W_i = A_i U_i = \bar{A}_i \bar{u}_i L_i^{\alpha-\beta}$$

  - Spatial equilibrium implies that workers keep migrating until $W = W_i$

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$$W_i = A_i U_i = \bar{A}_i \bar{u}_i L_i^{\alpha-\beta}$$

- Critical assumption: congestion forces dominate agglomeration forces
  - Need this in all spatial models
  - In Rosen-Roback it translates into an easy parameter restriction: $\beta > \alpha$
  - Implies: no uninhabited locations unless $\bar{U}_i = 0$ or $\bar{A}_i = 0$, as $L_i \to 0, W_i \to \infty$
- But we need a final equation to solve for the value W

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- First rewrite the spatial equilibrium condition as follows:

$$L_i = (\frac{W}{\bar{A}_i \bar{U}_i})^{\frac{1}{\alpha - \beta}}$$

- The final quation is then population adding-up constraint:

$$\bar{L} = \sum_i L_i = \sum_i (\frac{W}{\bar{A}_i \bar{U}_i})^{\frac{1}{\alpha - \beta}} \Rightarrow W = [\bar{L}^{-1}(\sum_i (\bar{A}_i \bar{U}_i)^{\frac{1}{\beta - \alpha}})]^{\beta - \alpha}$$

- Using this we can solve for the distribution of workers in terms of parameters:

$$L_i = \frac{(\bar{A}_i \bar{U}_i)^{\frac{1}{\beta - \alpha}}}{\sum_i (\bar{A}_i \bar{U}_i)^{\frac{1}{\beta - \alpha}}} \bar{L} \Rightarrow \frac{L_i}{\bar{L}} \equiv \pi_i = \frac{(\bar{A}_i \bar{U}_i)^{\frac{1}{\beta - \alpha}}}{\sum_i (\bar{A}_i \bar{U}_i)^{\frac{1}{\beta - \alpha}}}$$

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- The equilibrium allocation is then pinned down by two equations
- Labor+Goods market clearing:

$$w_i = Ai$$

  - Follows from firm optimization and worker optimal consumption decision
- Spatial Equilibrium:

$$L_i = \frac{(\bar{A}_i \bar{U}_i)^{\frac{1}{\beta - \alpha}}}{\sum_i (\bar{A}_i \bar{U}_i)^{\frac{1}{\beta - \alpha}}} \bar{L} \equiv \pi_i \bar{L}$$

  - Follows from optimal location decisions

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- Some notes:
  - The population and wage distributions fully characterizes the equilibrium
  - In spatial equilibrium, there are nominal wage differences across space exist
  - In spatial equilibrium, real wage differences across space reflect amenity differences
  - In spatial equilibrium any local improvement (productivity or amenities) leads to a welfare increase for all workers everywhere
  - Individual workers would be willing to change location immediately if asked

CALIBRATING THIS MODEL

- In spatial models there are two types of objects to calibrate:
  - Parameters, e.g., β, α
  - Location Fundamentals, e.g., $\bar{A}_i, \bar{U}_i$
- To identify parameters we require exogenous variation or a method of moments.
- Location fundamentals inferred as "structural residuals" conditional on parameters.
  - This yields the tight connection between data and model in these models

CALIBRATING THIS MODEL

- Suppose we know the deep parameters α and β
  - We can then exploit equilibrium relationships to identify fundamentals
- Step 1:
  - We know $A_i = w_i$, therefore $\bar{A}_i = w_i L_i^{-\alpha}$ (RHS = Data!)
- Step 2:
  - We use $L_i/\bar{L} = (\bar{A}_i \bar{U}_i)^{\frac{1}{\beta-\alpha}} [\sum_i (\bar{A}_i \bar{U}_i)^{\frac{1}{\beta-\alpha}}]^{-1}$, with inferred $\bar{A}_i$ and population shares data to infer $\bar{U}_i$

CALIBRATING THIS MODEL

- The first two steps highlights: the model can match data on wages and population shares exactly, for any choice of deep parameters α, β.
- This simple model does not provide enough restrictions to estimate α, β but in general this is possible using IV regression implied by the model, e.g.:

$$\log(w_i) = \alpha \log(L_i) + \log(\bar{A}_i) \Rightarrow \Delta \log(w_i) = \alpha \Delta \log(L_i) + \Delta \log(\bar{A}_i)$$

- To identify α need a population inflow that is unrelated to growth in local fundamental productivity, e.g., a population expulsion somewhere else, see Peters 2020.

WRITING THIS MODEL IN CHANGES (LIKE JONES 1965)

- Write the wage equation in changes:

$$\hat{w}_i = \hat{A}_i$$

- Write the population share equation in changes:

$$\hat{\pi}_i = \frac{(\hat{\bar{A}}_i \hat{\bar{U}}_i)^{\frac{1}{\beta - \alpha}}}{\sum_i \pi_i (\hat{\bar{A}}_i \hat{\bar{U}}_i)^{\frac{1}{\beta - \alpha}}}$$

- To solve for the impact of population shares and wages to a 10% increase in amenities in location i, just plug in $\hat{\bar{U}}_i = 1.1$ and $\hat{\bar{A}}_i = 1$ alongside *data* for population shares in original equilibrium, π_i

ALLEN ARKOLAKIS

SETUP

- Set of discrete location $i \in S$
- Locations differ in local amenities U_i and productivities A_i
  - These could potentially be a function of local population with some elasticity
- Total mass of workers $\bar{L}$ in the economy, choose locations to maximize utility
- Armington structure: each location produces its own variety, perfect competition, constant returns to scale
- Iceberg trade costs between locations

CONSUMERS

- Consumers have CES preferences over all regional varieties. Total utility:

$$V_j = \left(\sum_{i \in S} q_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \times U_j$$

- This implies indirect utility in location j is given by:

$$W_j = \frac{w_j}{P_j} U_j$$

- where P_j is the standard CES price index.

- Consumer choose locations so that $i^\star = \arg\max_i \{W_i\}$

PRODUCTION

- Production is as before in the Armington model: labor only, CRS:

$$y_i = A_i L_i$$

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- We can solve the equilibrium using the following three conditions:
  - 1. Goods+Labor market clearing yields:

$$w_i L_i = \sum_j \lambda_{ij} w_j L_j$$

  - 2. Spatial Equilibrium: there exists a W such that for all $i \in S$ such that $L_i > 0$, $W_i = W$ and for all $i \in S$ such that $L_i = 0$, $W_i \leq W$.
  - 3. Population Adding up: local labor supply sums to total world population

$$\sum_{i \in S} L_i = \bar{L}$$

INFERRING REGIONAL FUNDAMENTALS

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- We can write the goods+labor market clearing equation as follows:

$$w_i^\sigma L_i = W^{1-\sigma} \sum_{j \in S} \tau_{ij}^{1-\sigma} A_i^{\sigma-1} U_j^{\sigma-1} w_j^\sigma L_j$$

- where we used the spatial equilibrium condition that $W_i = W$
- From the spatial equilibrium equation itself we get:

$$w_i^{1-\sigma} = W^{1-\sigma} \sum_{j \in S} \tau_{ji}^{1-\sigma} U_i^{1-\sigma} A_j^{\sigma-1} w_j^{1-\sigma}$$

- These are two linear equations in $w_i^\sigma L_i$ and $w_i^{1-\sigma}$

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- We can write this system as follows:

$$\mathrm{x} = \lambda \mathrm{A}\mathrm{x} \quad \mathrm{y} = \lambda \mathrm{A}^T \mathrm{y}$$

- where $x_i = w_i^\sigma L_i$ and $y_i = w_i^\sigma$ and $\lambda = W^{1-\sigma}$.
- As long as $A_{ij} > 0$ Perron-Frobenius theorem guarantees there exist strictly positive, to-scale vectors x and y corresponding to the largest eigenvalue λ
- Since kernels of the system are transposes, eigenvalues are the same.
- Scale of wages is arbitrary (pinned down by choice of numeraire), scale of population is pinned down by the world population constraint

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- Why was this easy? There were enough dispersion forces in the system!
  - Armington assumption acts like dispersion force, since wages will go to infinity if no one is in a location.
  - Absence of agglomeration forces in the basic Armington model
    - Balance of dispersion and agglomeration forces sufficiently tilted toward dispersion!
- More difficult to prove existence in model with external effects in productivities and/or amenities: need restrictions on the strength of agglomeration forces.

FROM ALLEN ARKOLAKIS (2014): PROPERTIES OF MODEL WITH SPILLOVERS

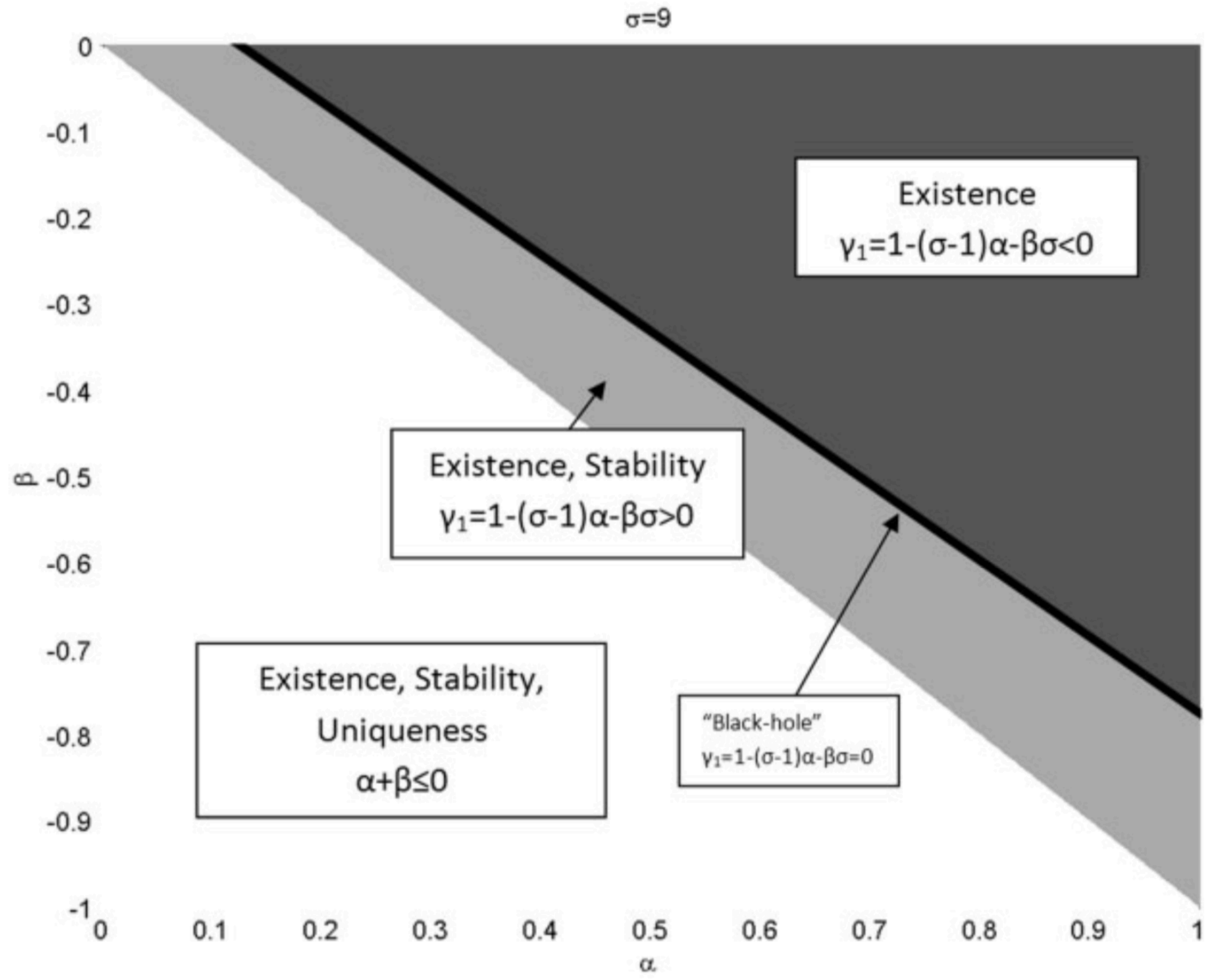

FIGURE I

Equilibria with Amenity and Productivity Spillovers

This figure shows the regions of values for the productivity spillover α and the amenity spillover β for which there exists an equilibrium, for which there exists a point-wise locally stable equilibrium, and whether that equilibrium is unique. The elasticity of substitution σ is chosen to equal 9.

MORE LABOR SUPPLY MODULES

FREE MOBILITY

- So far we saw the two easiest of labor supply modules:
  - *No Mobility*: Labor supply in each location is fixed to a constant $L_i = \bar{L}_i$
  - *Free Mobility*: Labor supply in each location adjusts so that indirect utilities are equalized across locations, $W_i = W \, \forall i$ s.t. $L_i > 0$
- Next we look at how to generate the in-between cases: local labor supply responds to increases in local wages, rents, or fundamentals but less than one-for-one.
  - We say local labor supply is *upward sloping* (in wages)

FRÉCHET SHOCKS

- We introduce an idiosyncratic element into agent js utility:

$$W_i^j = f(w_i, P_i, U_i)\xi_i^j$$

- We assume that agent j receives an *iid* idiosyncratic amenity draw ξ_i^j for *each* location i before making their decision.

- Agent j hence solves the following problem:

$$i^\star = \max_i \{f(w_1, P_1, U_1)\xi_1^j, \ldots, f(w_S, P_S, U_S)\xi_S^j\}$$

FRÉCHET SHOCKS

- But this looks a lot like the problem we saw in Eaton and Kortum (2002): discrete choice problem with stochastic element!
- Let's assume that

$$F(\xi_i^j) = \exp(-z^{-\theta}) \quad \theta > 1$$

- This gives the following analytic expression for the fraction of workers that choose to live in i:

$$\frac{L_i}{\bar{L}} = \pi_i = \frac{(f(w_i, P_i, U_i))^\theta}{\sum_i (f(w_i, P_i, U_i))^\theta}$$

FRÉCHET SHOCKS: ALTERNATIVE

- Suppose the Frechet distributions for each destination has a non-unitary mean:

$$F(\xi_i^j) = \exp(-T_i z^{-\theta}) \quad \theta > 1$$

- This gives the following analytic expression for the fraction of workers that choose to live in i:

$$\frac{L_i}{\bar{L}} = \pi_i = \frac{T_i(f(w_i, P_i, U_i))^\theta}{\sum_i T_i(f(w_i, P_i, U_i))^\theta}$$

- But then the T_i is the "mean amenity" and isomorphic to U_i, so drop U_i.
  - Alternative ways to think about amenities

FRÉCHET SHOCKS: LABOR SUPPLY ELASTICITY

- Consider embedding this into the Armington model above.
  - Then $f(\cdot) = (w_i/P_i)U_i$
- It is then easy to see that the labor supply elasticity in location i is:

$$\frac{w_i}{\pi_i}\frac{\partial \pi_i}{\partial w_i} = (1 - \pi_i)\theta$$

- where we assumed that $\partial P_i/\partial w_i = 0$.
- So θ now governs local labor supply, if its large local employment responds strongly to local increases in the wage

FRÉCHET SHOCKS: UTILITY ACROSS SPACE

- Workers are now no longer indifferent across regions!
- However, ex-ante, before they receive their idiosyncratic shocks they all have the same expected value:

$$\bar{W} = (\sum_i (f(w_i, P_i, U_i))^\theta)^{\frac{1}{\theta}}$$

- Ex-post, they individuals are not indifferent.
  - However: *distribution* of utilities within each location is the same:
  - More productive regions have more workers, and lower average idiosyncratic utility draws!
  - There is now a marginal worker, that will move if something changes

LIMIT CASES: PREFERENCE SHOCKS AS MIGRATION FRICTIONS

- As the dispersion of idiosyncratic preference shocks $(1/\theta)$ goes to infinity:
$$\lim_{\theta \to 0} \pi_i = \lim_{\theta \to 0} \frac{T_i(f(w_i, P_i))^\theta}{\sum_i T_i(f(w_i, P_i))^\theta} = \frac{T_i}{\sum_i T_i}$$
- Its all about preferences in the limit, wages and prices play no role!
  - Like migration frictions: prevent closing of real wage gaps across space!
- As dispersion goes to zero, agents agree only care about differences in real wages, location with highest real wage gets all workers!
  - Like free mobility case: in equilibrium no real wage gaps across space

GUMBEL SHOCKS

- Sometimes it is more convenient to have the idiosyncratic preferences enter linearly:

$$W_i^j = f(w_i, P_i, U_i) + \xi_i^j$$

- Assume ξ_i^j is drawn iid from a Gumbel distribution $F(\xi_i^j) = \exp\{-\exp\{-\xi\theta\}\}$
- Then going again to math similar to Eaton and Kortum (2002):

$$\frac{L_i}{\bar{L}} = \pi_i = \frac{\exp\left(\theta f(w_i, P_i, U_i)\right)}{\sum_i \exp\left(\theta f(w_i, P_i, U_i)\right)}$$