INTERNATIONAL TRADE - ECON 245

FABIAN ECKERT

HETEROGENEOUS FIRMS II

INTRODUCTION

- Another famous model with heterogeneous firms is Eaton and Kortum (2002).
- Different from Melitz (2003) [IRS+MC] EK features CRS+perfect competition
- The model is a probabilistic generalization of the Ricardian model by DFS to capture competing forces:
  - Comparative advantage differences promote trade
  - Geographic barriers (transport costs, tariffs etc) diminish trade
- The probabilistic formulation itself is a hugely influential technical contribution

MODEL SETUP

- Countries are indexed by $i \in 1,\ldots,N$
- Continuum of goods $\omega \in [0,1]$
- Labor-only production.
- Labor mobile across industries, but immobile across countries
- Constant Returns to Scale production with $z_i(\omega)$ productivity for good ω
- The usual iceberg trade costs τ_{ij} with triangle inequality holding ($\tau_{ik}\tau_{kj} > \tau_{ij}$)

PREFERENCES

- Consumers have CES preferences over the set of industries $\omega \in [0,1]$
- Goods of each industries are homogeneous across countries
- Market structure is perfect competition, so prices are given by marginal cost:

$$p_{ij}(\omega) = \frac{w_i}{z_i(\omega)}\tau_{ij}$$

- Consumer expenditure minimization implies that they buy from the country offering the lowest price. So the price paid by consumers in destination j:

$$p_j(\omega) = \min_{i=1,\ldots,N} \{p_{ij}(\omega)\}$$

TECHNOLOGY

- Country i's efficiency in producing good ω is the realization of a random variable (drawn iid for each good and country) Z_i drawn from $F_i(z) = Pr(Z_i \leq z)$
  - By LLN $F_i(z)$ is fraction of goods for which country i has efficiency below z
- Eaton and Kortum choose F to be the following Fréchet distribution:

$$F_i(z) = \exp(-T_i z^{-\theta}) \quad T_i > 0 \quad \theta > 1$$

  - The parameter T_i captures absolute advantage of country i
  - The parameter θ is a (inverse) measure of the degree of comparative advantage

ASIDE: A USEFUL PROPERTY OF EXTREME VALUE DISTRIBUTIONS

- Distributions in the class of *extreme value distributions* are “max and min stable”
  - E.g.: Fréchet, Pareto, Gumbel, multivariate version of those
- These distributions have a key property useful to economists:
  - E.g.: The minimum or maximum of a list of iid-Fréchet variables again itself follows a Fréchet distribution:

$$X_{min} = \min\{x_1, x_2, \ldots, x_N\} \quad \text{where} \quad x_i \sim \text{Frechet} \quad \text{then} \quad X_{\min} \sim \text{Frechet}$$

PRICES

- The origin country i presents a destination j with a distribution of prices $G_{ij}(p) = Pr[p_j \leq p] = 1 - F_i(w_i\tau_{ij}/p)$:

$$G_{ij}(p) = 1 - \exp\left(-\left[T_i\left(w_i\tau_{ij}\right)^{-\theta}\right]p^{\theta}\right)$$

- The distribution of the minimum of prices (i.e. actual consumer prices) in destination j, $G_j = Pr[P_j \leq p]$ is

$$G_j(p) = 1 - \prod_{i=1}^{N}[1 - G_{ij}(p)]$$

- Substituting yields

$$G_j(p) = 1 - \exp(-\Phi_j p^{\theta}) \quad \text{where} \quad \Phi_j \equiv \sum_i^N T_i(w_i\tau_{ij})^{-\theta}$$

COROLLARIES OF THE FRÉCHET ASSUMPTION

- The probability country i provides a given good at the lowest price to j is

$$Pr[p_{ij} \leq \min_{k \neq i}\{P_{kj}\} \equiv \pi_{ij} = \frac{T_i(w_i\tau_{ij})^{-\theta}}{\Phi_i} = \frac{T_i(w_i\tau_{ij})^{-\theta}}{\sum_k T_k(w_k\tau_{kj})^{-\theta}}$$

  - by the LLN this is also the fraction of goods j buys from i

- The price of a good that country n actually buys from any country i also has the distribution $G_i(p)$

  - Source which is more productive/closer/cheaper sells *more goods* to the point where the distribution of prices is the same as for every other source!

COROLLARIES OF THE FRÉCHET ASSUMPTION

- With CES utility and a Fréchet distribution for the prices actually paid by consumers, the price index takes the following form:

$$P_j = \Gamma \Phi_j^{-1/\theta} \quad \text{where} \quad \Gamma \equiv [\Gamma(\frac{\theta + 1 - \sigma}{\theta})]^{\frac{1}{1-\sigma}}$$

  - Eaton+Kortum derive this using moment generating function
    - Sketch: $E(e^{tx}) = \Phi_j^{\frac{t}{\theta}} \Gamma(1 - \frac{t}{\theta}) \quad x = -\ln p \quad E(p^{-t})^{-\frac{1}{t}} = \Gamma(1 - \frac{t}{\theta})^{-\frac{1}{t}} \Phi^{-\frac{1}{t}}$
  - Γ is the so called Gamma function (check on Wikipedia)
  - We impose the parameter restriction $\sigma < 1 + \theta$ for the index to be well defined
  - Φ_j summarizes technology, inputs costs, and geographic barriers around the world

TRADE FLOWS AND GRAVITY

- Price distribution sourced from each country is invariant to origin
  - All of the adjustment in trade flows is in the extensive margin of the mass of goods, not the intensive margin of average expenditure per good
- As a result, the fraction of goods sourced from an origin is also the fraction of spending!

$$\pi_{ij} = \lambda_{ij} = \frac{X_{ij}}{X_j} = \frac{T_i(w_i\tau_{ij})^{-\theta}}{\sum_k T_k(w_k\tau_{kj})^{-\theta}}$$

- Notice that total sales in the origin are:

$$Q_i = \sum_j X_{ij} = T_i w_i^{-\theta} \sum_j \frac{\tau_{ij}^{-\theta} X_j}{\Phi_j}$$

TRADE FLOWS AND GRAVITY

- We can solve the expression for Q_i for the $T_i w_i^{-\theta}$ and substitute into the trade shares to obtain:

$$X_{ij} = \frac{(\tau_{ij}/p_i)^{-\theta} X_i}{\sum_k (\tau_{kj}/p_k)^{-\theta} X_k}$$

- The gravity equation captures bilateral and multilateral resistance
- The numerator captures market size of a given destination as perceived from origin; the denominator total market size/access of origin
- Since more productive/close/cheap exporters expand on extensive margin the trade elasticity depends on θ not on σ

ESTIMATING θ

- Dividing two trade share expressions we derive

$$\frac{\lambda_{ij}}{\lambda_{jj}} = \frac{\Phi_j}{\Phi_i}\tau_{ij}^{-\theta} = (\frac{P_j\tau_{ij}}{P_i})^{-\theta}$$

- The LHS is a normalized import share in j
- Eaton and Kortum then graph the relative import shares against distance
  - Suggests one way of estimating θ

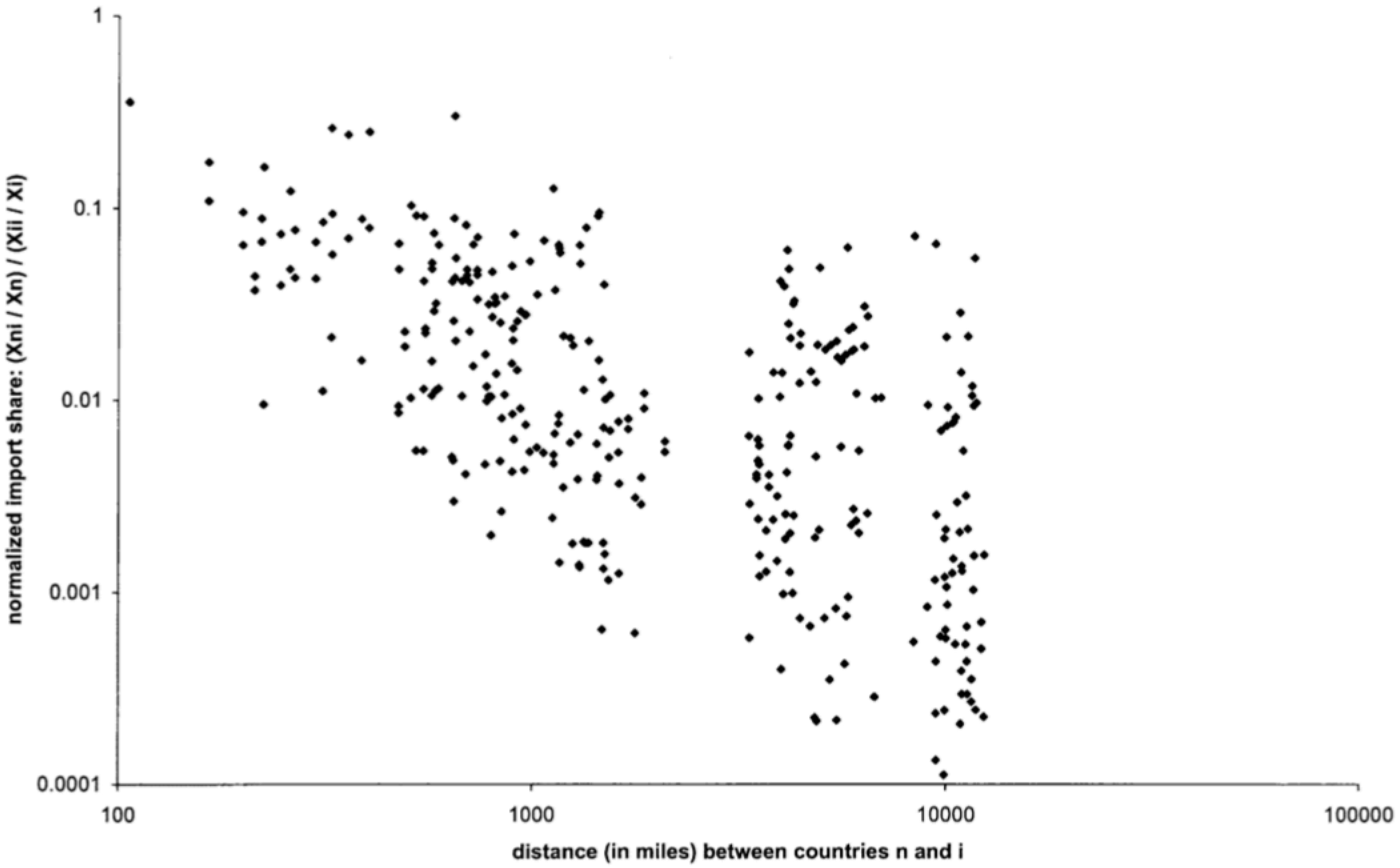

FIGURE 1.—Trade and geography.

EQUILIBRIUM

- The equilibrium is then computed exactly as in the Armington model.
- All endogenous objects can be expressed as a function of $\{w_i\}$ only.
- Goods market clearing implies:

$$w_i L_i = \sum_j \lambda_{ij} w_i L_i \quad \text{where} \quad \sum_i \lambda_{ij} = 1$$

- Simple iterative procedure will solve efficiently for $\{w_i\}$

WELFARE AND GAINS FROM TRADE

- We again obtain an elegant GFT expression: $\hat{W} = \lambda_{ii}^{-\frac{1}{\theta}}$
- Country size influence gains from trade through λ_{ii}.
  - Gains a greater the more heterogeneity in efficiency (the smaller θ)
- In conclusion: the modern trade models we saw (Krugman, Armington, Melitz, Eaton+Kortum)
  - …all yield gravity equations for bilateral trade flows and
  - …all have a similar formula for gains from trade