

INTERNATIONAL TRADE - ECON 245

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**EXTENSIONS OF
THE CANONICAL SPATIAL MODEL**

INTRODUCTION

- ▶ The workhorse “quantitative spatial model” is modular:
 - ▶ “Modules” for producers, consumer, and market structure can be combined arbitrarily
 - ▶ Each “modules” introduces new parameters and new regional fundamentals
 - ▶ Fundamentals can always be inferred as structural residuals
- ▶ **Overall:** flexible modeling framework for any quantitative spatial question

PRODUCTION MODULES REVIEW

- ▶ We saw four ways to specify the production side:
 - ▶ *Rosen-Roback*: homogeneous good produced under perfect competition
 - ▶ *Armington*: region-specific variety produced under perfect competition with CES “love for variety” preferences
 - ▶ *Eaton Kortum*: Continuum of goods; each region/country has probabilistic productivity at producing each; perfect competition+CES “love for variety” preferences
 - ▶ *Krugman*: free entry of firms (varieties) under monopolistic competition with CES “love for variety” preferences

PLAN FOR TODAY

- ▶ Today will cover a range of important additional “modules:”
 - ▶ Skill types
 - ▶ Sectors
 - ▶ Rental markets
 - ▶ Commuting
 - ▶ Input-Output Linkages

GENERAL SETUP

- ▶ For convenience assume the same setup throughout (unless otherwise stated)
- ▶ Discrete set of N regions
 - ▶ Armington economy with region-specific varieties and perfect competition
 - ▶ CES preferences over regional varieties
 - ▶ Index locations by i, j , individual workers by ω
 - ▶ Frechet shocks for location choice, with inverse variance θ and mean 1

SKILL

HETEROGENEITY

INTRODUCTION

- ▶ Introduce skill types k ; for simplicity consider case with high ($k = h$) and low ($k = l$)
- ▶ Two ways to make skill enter the canonical spatial model
 1. Worker types enter production function symmetrically
 - ▶ Firm production function: $y_i = A_i h_i$ where h_i are *efficiency units of labor* supplied by *either* high- or low-skill workers
 2. Worker types enter production function differently
 - ▶ CES production function with two skill types: $y_i = \left(\alpha_h l_{i,h}^{\frac{\rho-1}{\rho}} + \alpha_l l_{i,l}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$

EFFICIENCY UNITS OF LABOR

- ▶ Workers differ in their efficiency units of labor; w_i wage per efficiency unit
- ▶ Worker ω can provide h^ω units of efficiency labor
 - ▶ Workers draw their efficiency units from a Fréchet distribution *after* choosing locations

$$F_k(h) = \exp(-T_k h^{-\vartheta}) \quad \text{where} \quad T_k > 0 \quad \text{and} \quad \vartheta > 1$$

- ▶ If $T_h > T_l$ the average high-skill worker supplies more labor units
 - ▶ The average wage among workers in each group: $\bar{w}_{i,k} = T_k^{1/\vartheta} w_i$ (via Fréchet math)
 - ▶ The skilled wage premium is then simply $\bar{w}_{i,h}/\bar{w}_{i,l} = (T_h/T_l)^{1/\vartheta}$

EQUILIBRIUM SKETCH

- ▶ The local labor/goods market clearing condition in efficiency units:

$$H_i w_i = \sum_j \lambda_{ij} H_j w_j$$

where we can show that $H_{i,k} = L_{i,k} \Gamma(\theta) T_i^{1/\theta}$ and $H_i = H_{i,h} + H_{i,l}$ (Frechet math)

- ▶ Separate location choice equations for high and low skill agents:

$$L_{i,k} = \frac{\bar{w}_{i,k}^\theta}{\sum_i \bar{w}_{i,k}^\theta} \bar{L}_k$$

Expected wages due to uncertainty about skill shocks, received after moving

AN ALTERNATIVE OR ADDITION: CES PRODUCTION FUNCTION

- ▶ With CES formulation can either work in bodies or efficiency units

$$\frac{w_{i,h}}{w_{i,l}} = \left(\frac{l_{i,h} \alpha_l}{l_{i,l} \alpha_h} \right)^{-1/\rho} \quad \text{or} \quad \frac{w_{i,h}}{w_{i,l}} = \left(\frac{h_{i,h} \alpha_l}{h_{i,l} \alpha_h} \right)^{-1/\rho}$$

Attractive: local skill premium now depends on relative local labor supply

- ▶ Now have *separate* local labor markets for high and low skill workers
 - ▶ Use skill premium expression to write goods/labor market clearing just in terms of high- or low-skill wage

A NOTE ON SKILL TYPE SPECIFIC AMENITIES

- ▶ With CES *production* function local factor supply determines local skill premium
- ▶ Locations may differ in the amenities they provide to different skill groups.
- ▶ Consider the indirect utility of a type k worker: $W_{i,k} = u_{i,k} \frac{\bar{w}_{i,k}}{P_i}$
- ▶ With free mobility $W_{i,k} = \bar{W}_k$, all else equal high amenities entail low wages
- ▶ High-skill workers are cheap in New York since it has high skill amenities
 - ▶ Part of New York's comparative advantage in producing skill-intensive goods

SECTORS

INTRODUCTION

- ▶ The one-sector Armington model is designed to justify *intraindustry* trade
- ▶ The Ricardian and Heckscher-Ohlin models explained *sectoral* specialization
- ▶ We now have the tools to re-introduce sectors into the spatial model
 - ▶ Allow for both technology (Ricardian) and factor endowment differences (Heckscher Ohlin) motives for sectoral specialization
 - ▶ With Armington setup: never complete specialization, but always net importer in at least one sector, and net exporter in another

INTRODUCING SECTORS

- ▶ Regions have two sectors s ; each produce a unique Armington variety
- ▶ Consumer spend fraction α_s on sectors s Armington bundle
- ▶ Locations differ in the sector-specific productivities $A_{r,s}$ (Ricardo!)
- ▶ Each sector requires efficiency units of labor to produce:

$$y_{r,s} = A_{r,s} h_{r,s}$$

- ▶ Efficiency units can be supplied by high- or low-skill workers alike
 - ▶ Wage per efficiency unit ($w_{r,s}$) differs across sectors, not skill types since workers are perfectly substitutable in production function, but not across sectors

SECTORS AND SKILLS

- ▶ Within each skill group, workers differ in their productivity for each sector
 - ▶ *Idiosyncratically*: some workers are better in some sector
 - ▶ *Systematically*: high-skill workers are better in some sector
- ▶ Worker draw the efficiency units of labor they can supply from Fréchet distribution:

$$F_{s,k}(h) = \exp(-T_{s,k}h^{-\vartheta}) \quad T_{s,k} > 0 \quad \vartheta > 1$$

- ▶ Skilled workers have an *absolute advantage* $T_{h,s} > T_{l,s} \quad \forall s$
- ▶ Workers have a *comparative advantage* in one sector $T_{h,1}/T_{h,2} > T_{l,1}/T_{l,2}$

SECTOR CHOICES AND SECTORAL LABOR SUPPLY IN EFFICIENCY UNITS

- ▶ The fraction of workers of each skill type that choose a given sector:

$$\phi_{i,k,s} = \frac{T_{k,s}(w_{i,s})^\vartheta}{\sum_{s'} T_{k,s}(w_{i,s'})^\vartheta} \quad \text{and} \quad H_{i,k,s} = L_{i,k} \Gamma(\vartheta) T_{k,s}^{1/\vartheta} \phi_{i,k,s}^{\frac{\vartheta-1}{\vartheta}}$$

- ▶ Holding wages equal:

- ▶ If $T_{h,1}/T_{h,2} > T_{l,1}/T_{l,2}$ larger fraction of h workers choose $s = 1$ than l workers
- ▶ Comparative advantage drives sector choice; absolute advantage irrelevant!

WAGE DIFFERENCES ACROSS SKILL GROUPS

- ▶ “Fréchet math” yields an expression for the average wage by skill type

$$\bar{w}_{i,k} = \left(\sum_s T_{k,s} w_{i,s}^\vartheta \right)^{1/\vartheta}$$

- ▶ With $T_{h,s} > T_{l,s} \forall s$, $\bar{w}^h > \bar{w}^l$: absolute advantage drives wage level differences
- ▶ Worker type k more exposed to wages changes in sector of comparative advantage
- ▶ Skill premium $\bar{w}_{i,h}/\bar{w}_{i,l}$ a function of relative supply and sectoral choices even without CES!

EQUILIBRIUM

- ▶ Labor/Goods market clearing equation for each sector:

$$w_{i,s}H_{i,s} = \sum_{j,s'} \lambda_{ij}^s \alpha_s w_{j,s'} H_{j,s'}$$

where α_s is spending share on sector s and $H_{i,s}$ total efficiency units in i, s

- ▶ Optimal sectoral choices:

$$\phi_{i,k,s} = \frac{T_{k,s}(w_{i,s})^\vartheta}{\sum_{s'} T_{k,s'}(w_{i,s'})^\vartheta} \quad \text{and} \quad H_{i,k,s} = L_{i,k} \Gamma(\vartheta) T_{k,s}^{1/\vartheta} \phi_{i,k,s}^{\frac{\vartheta-1}{\vartheta}} \quad \text{and} \quad H_{i,s} = \sum_k H_{i,k,s}$$

- ▶ Optimal Location choices ($u_{i,k}$: Heckscher Ohlin element!)

$$L_{i,k} = \frac{u_{i,k}(\bar{w}_{i,k})^\theta}{\sum_i u_{i,k}(\bar{w}_{i,k})^\theta} \bar{L}_k \quad \text{where} \quad \bar{w}_{i,k} = \left(\sum_s T_{k,s} w_{i,s}^\vartheta \right)^{1/\vartheta}$$

HORSERACE BETWEEN RICARDO AND HECKSCHER-OHLIN

- ▶ Suppose there are two sectors tradable services ($s = TS$) and goods ($s = TG$)
- ▶ Then locations that have $A_{i,TS} > A_{i,TG}$ would tend to specialize in goods
- ▶ And high skill workers would tend to work in TS if $T_{h,TS}/T_{h,TG} > T_{l,TS}/T_{l,TG}$
- ▶ However:
 - ▶ If a region with $A_{i,TS} > A_{i,TG}$ has very high low skill amenities, so that low-skill workers are cheap it may still be a net exporter of TG
 - ▶ Horse-race between Heckscher-Ohlin and Ricardo motives

RENTAL MARKETS

INTRODUCTION

- ▶ A distinctly *spatial* feature of the economy are housing markets
 - ▶ They clear locally, in contrast to goods markets
 - ▶ Their supply is constrained by local geological features or laws
 - ▶ They are key ingredient in differences in cost of living across locations
 - ▶ They are a natural “congestion” force in spatial models
 - ▶ They are central in regulating access to local amenities and labor markets

PREFERENCES

- ▶ The easiest way to introduce housing is a nested structure
 - ▶ Cobb Douglas utility with $\alpha \in (0,1)$ the expenditure share on the traded CES bundle
 - ▶ The resulting indirect utility from locating in location i is then given:

$$W_i = \frac{w_i}{P_i^\alpha r_i^{1-\alpha}}$$

HOUSING SUPPLY: FIXED

- ▶ The simplest way is to assume a fixed housing supply H_i in each location
- ▶ This changes the good/labor market clearing equation:

$$w_i L_i = \sum_j \lambda_{ij} w_j L_j \alpha (1 + \phi)$$

- ▶ $(1 + \phi)$ explained shortly
- ▶ And it adds a local housing market clearing condition to solve for local rents:

$$(1 - \alpha)(1 + \phi)w_i L_i = r_i H_i$$

- ▶ α can be obtained from Consumer Expenditure Survey, rent data from Decennial Census or Zillow: can infer H_i as structural residual from housing market clearing!

HOUSING SUPPLY: ELASTIC

- ▶ In reality housing supply is likely to adjust if more families move into a location
- ▶ A reduced form way of modeling this to specific local housing supply as follows:

$$H_i = \bar{H}_i L_i^\psi \quad \text{where usually } \psi \in (0,1)$$

- ▶ The concavity in the supply models that as more and more families move in land becomes unavailable
- ▶ ψ can be estimated from relationship between Δr_i and ΔL_i using an IV strategy

HOUSING SUPPLY: MICROFOUNDED

- ▶ Each location has land area S_i which can be combined with labor to produce housing services/or develop the land to be inhabitable:

$$H_i = (l_i^H)^\beta S_i^{1-\beta} \Rightarrow H_i = (1 - \beta)L_i^\beta S_i^{1-\beta}$$

- ▶ where s_i is demand for land in the housing sector, arrow holds in equilibrium only
- ▶ Since S_i is a fixed factor: endogenously varying house price elasticities across locations!
- ▶ Be careful that labor/goods market clearing changes since there are now two sectors in the economy.

WHO OWNS HOUSING?

- ▶ A crucial issues is the ownership of housing: who gets the return on housing $r_i H_i$?
- ▶ Several ways of modeling this:
 - ▶ **Not a good idea**: reimburse housing income to all local agents since this distorts location choices
 - ▶ National portfolio of housing in which everyone owns a share proportional to their income:

$$\sum_i w_i L_i \phi = \sum_i r_i H_i \Rightarrow \phi = \frac{\sum_i r_i H_i}{\sum_i w_i L_i}$$

- ▶ The total income of an agent is then $(1 + \phi)w_i$

WHO OWNS HOUSING?

- ▶ Other ways of modeling housing ownership:
 - ▶ Introduce second type of agent which cannot move and doesn't rent housing themselves just consumes goods: landlords
 - ▶ So that $r_i H_i$ is simply spent in location i on tradable goods
 - ▶ Very few papers thinking seriously about ownership of housing by individuals!

HOUSE PRICE ELASTICITIES AS AN INSTRUMENT

- ▶ Albert Saiz in the QJE: “The Geographic Determinants of Housing Supply”
 - ▶ Satellite data on terrain elevation and presence of water bodies to precisely estimate the amount of developable land in U.S. metropolitan areas.
 - ▶ Shows directly that “most areas in which housing supply is regarded as inelastic are severely land constrained by their geography”
 - ▶ Takeaway: “Geography is a key factor in the contemporaneous urban development of the United States”
- ▶ Widely used as an instrument: exogenous variation in Δr_i given ΔL_i !

FURTHER READING

- ▶ Hsieh Moretti 2018 (AEJ Macro): housing market restriction reduce US economic growth by preventing workers moving into most fast growing locations
- ▶ Ganong and Shoag 2017 (JUE): housing market restrictions deter low-skill migration into high-paying cities
- ▶ Ahlfeldt, Redding, Sturm, Wolf 2015 (ECMA): the impact of the building of the Berlin wall on the rent price gradient in the city
- ▶ Couture Gaubert Handbury Hurst 2019: displacement of poor workers in big cities as income inequality drives up the prices of housing in certain locations

COMMUTING

INTRODUCTION

- ▶ Migration is not the only way to access a productive labor market
 - ▶ Commuting provide an additional means of access that avoids paying the high cost of rent typically associated with productive locations
- ▶ Commuting infrastructure is a huge policy concern
 - ▶ The United States is largely specialized in “car accessibility” whereas Europe has greater “transit” accessibility.
- ▶ Commuting also matters for labor market, elasticity of local employment to a labor demand shock is heterogeneous depending on commuting openness

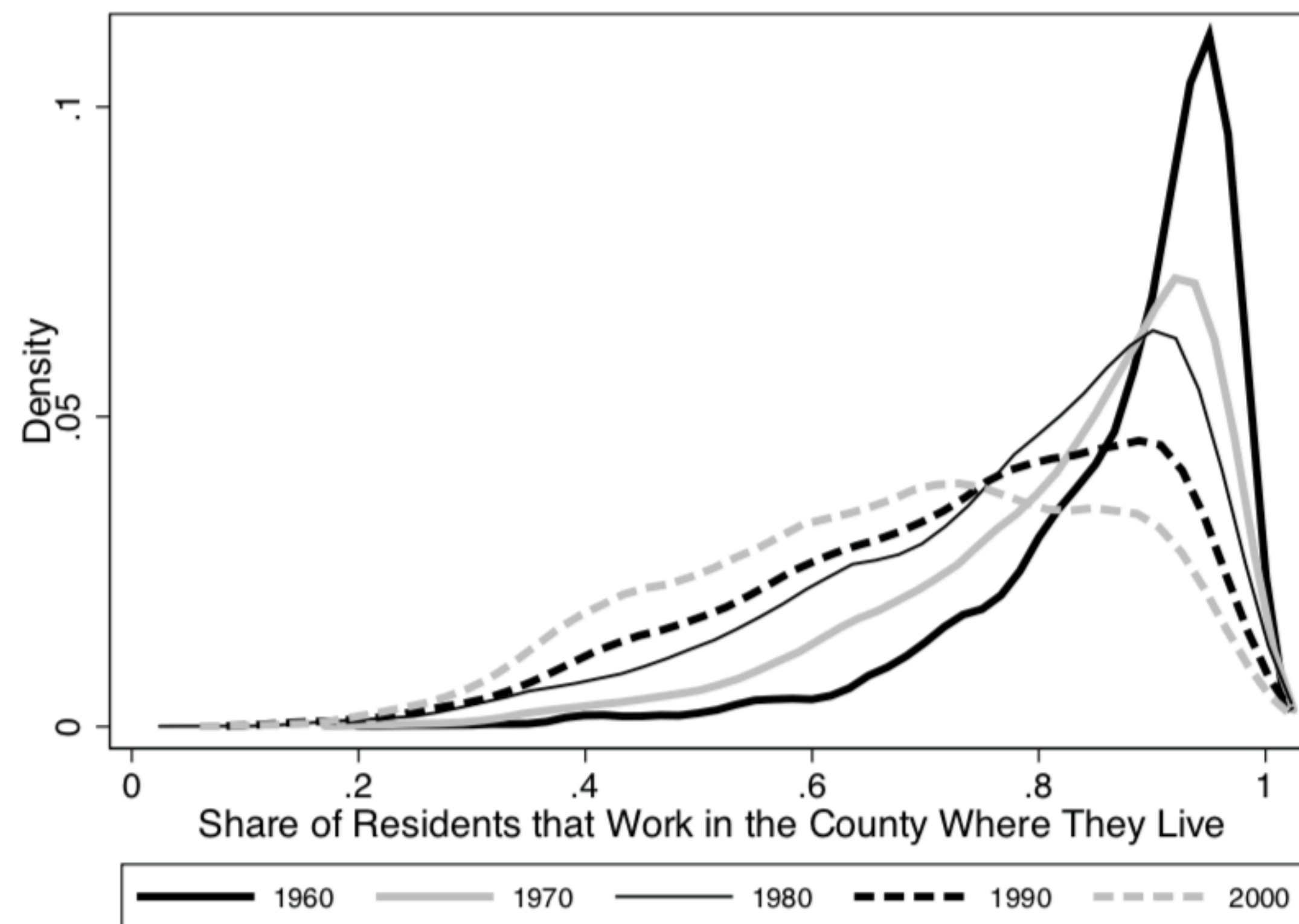
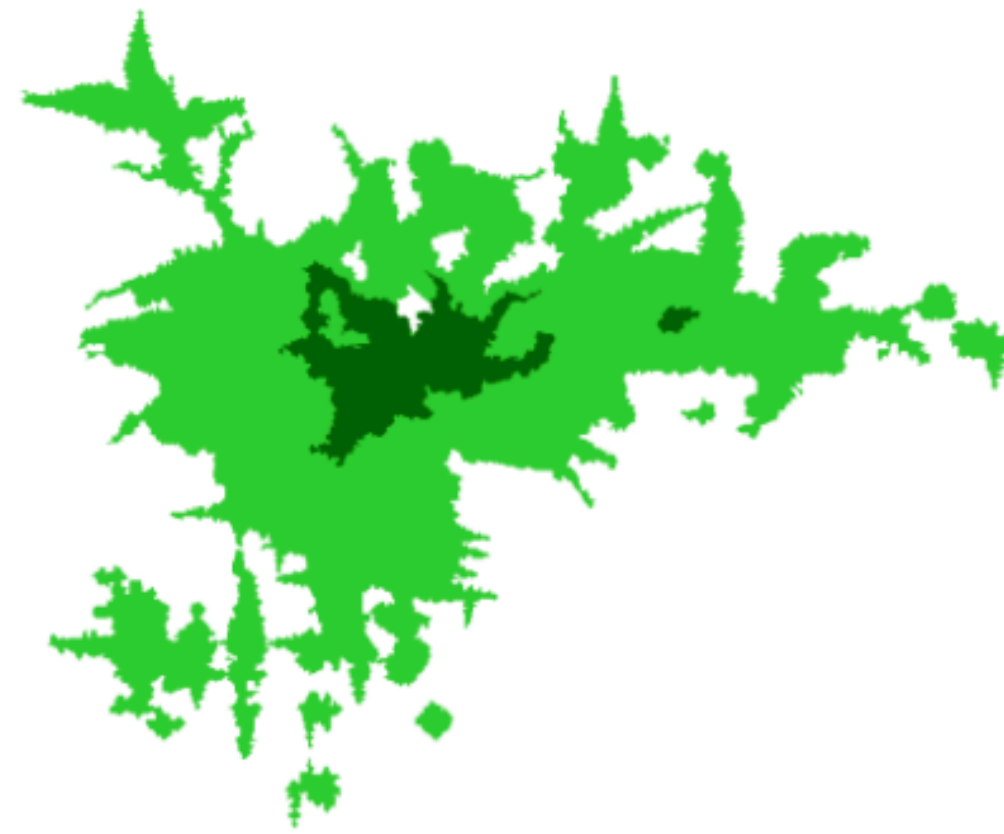


Figure 1: Kernel densities of the share of residents that work in the county where they live

FIGURE 1: PUBLIC TRANSIT ACCESSIBILITY ZONES
(30 MINUTES IN DARK, 60 MINUTES IN LIGHT)

Los Angeles, CA



London, UK



SETUP

- ▶ Cobb Douglas utility over a CES bundle of traded varieties, and local housing services
- ▶ Workers now choose *both* their location of residence and of work
 - ▶ Workers consume housing and amenities in the location in which they live
 - ▶ Workers command the wage associated with their location of work
 - ▶ Commuting cost incurred when working and residence location different
- ▶ Assume there are landlords in each location which own all land and spend all their income on the traded CES bundle

COMMUTING/LOCATION DECISION

- ▶ Worker ω derives the following indirect utility from living in i and working in j :

$$W_{ij}^{\omega} = \frac{\eta_{ij}^{\omega}}{\kappa_{ij}} \frac{w_j}{P_i^{\alpha} r_i^{1-\alpha}}$$

- ▶ Note i index on cost of living and j index on wages!
- ▶ Also:
 - ▶ η_{ij}^{ω} is an idiosyncratic preference shock for the ij combination
 - ▶ κ_{ij} is a commuting cost, so that $\kappa_{ii} = 1 \forall i$ and $\kappa_{ij} > 1 \forall i \neq j$

COMMUTING/LOCATION DECISION

- ▶ Workers than solve the the following problem:

$$\max_{ij} \{ W_{ij}^\omega \}$$

- ▶ To get convenient aggregation we assume η_{ij}^ω is Frechet distributed with mean 1 and inverse dispersion parameter θ .
- ▶ The fraction of workers making each residence-workplace decision:

$$\phi_{ij} = \frac{(\kappa_{ij} P_i^\alpha r_i^{1-\alpha})^{-\theta} w_j^\theta}{\sum_{i,j} (\kappa_{ij} P_i^\alpha r_i^{1-\alpha})^{-\theta} w_j^\theta} \equiv \frac{\Phi_{ij}}{\Phi}$$

EQUILIBRIUM

- ▶ Labor/Goods Market Clearing:

$$L_i^W w_i = \sum_j \lambda_{ij} \bar{w}_j L_j^R (1 + \phi) \alpha$$

where $\bar{w}_i = \sum_j [w_j \phi_{ij} / (\sum_j \phi_{ij})]$ is the average wage among i residents

- ▶ The number of workers and residents in each location is given respectively:

$$L_i^W = \sum_i \phi_{ij} \bar{L} \quad \text{and} \quad L_i^R = \sum_j \phi_{ij} \bar{L}$$

- ▶ Housing markets in each location clear: $L_i^R \bar{w}_i (1 - \alpha)(1 + \phi) = H_i r_i$

FURTHER READING

- ▶ Parro, Redding, Rossi-Hansberg 2018 (AER): introduce the extreme value commuting formulation
- ▶ Tsivanidis 2019 (JMP): studies the distributional effects of a rapid bus system in Bogota on workers' labor market access and firms' labor market access
- ▶ Severen (2021): housing market effects of mass transit infrastructure project in Los Angeles
- ▶ Ahlfeldt, Redding, Sturm, Wolf 2015 (ECMA): Impact of the building of the Berlin wall on location of economic activity in the city

INPUT-OUTPUT LINKAGES

INTRODUCTION

- ▶ So far all trade was in final goods – however trade in intermediate inputs is very important in the U.S. economy
- ▶ Gross output is almost double GDP (value added) in the U.S. economy
 - ▶ A lot of trade between US regions and in the world is hence intermediate input trade instead of trade in final goods/services
- ▶ Trade economists have developed a concise framework for thinking about input-output linkages between sectors and regions

PRODUCTION

- ▶ Suppose the firm production function in location i sector s is given by:

$$y_{i,s} = l_{is}^{\alpha_s} \prod_{s'} [(\sum_i q_{i,s'}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}]^{\gamma_{s's}(1-\alpha_s)} \quad \text{where} \quad \sum_{s'} \gamma_{s's} = 1$$

Value added share of output is α_s intermediate input share of sector s' is $\gamma_{s's}(1 - \alpha_s)$

- ▶ The price of the Armington variety produced in location i and sector s is given:

$$p_{i,s} = (w_{i,s}/A_{i,s})^{\alpha_s} \prod_{s'} P_{i,s'}^{\gamma_{s's}(1-\alpha_s)}$$

CLOSING THE MODEL

- ▶ Consumers have Cobb Douglas preferences *across sectors* and spend β_s on each sector s CES bundle of Armington varieties
- ▶ Workers choose locations subject to an idiosyncratic preference shock, and sectors subject to an idiosyncratic productivity shock
 - ▶ When choosing location workers take expectations over their sectoral shocks, i.e., sectoral productivity shocks realized after migration decision

EQUILIBRIUM

- ▶ Labor/Goods market clearing:

$$H_{i,s}w_{i,s} = \alpha_s \sum_j \lambda_{ij}^s [\beta_s (\sum_s H_{j,s}w_{j,s}) + (\sum_{s'} \gamma_{ss'}(1 - \alpha_s)Y_{j,s'})]$$

Note how workers receive only α_s share of total spending on the sector

- ▶ Sector choice equation:

$$L_{i,s} = \frac{T_s(w_{i,s})^\vartheta}{\sum_{s'} T_{s'}(w_{i,s'})^\vartheta} L_i$$

- ▶ Location choice equation:

$$L_i = \frac{\bar{w}_i^\theta}{\sum_i \bar{w}_i^\theta} \bar{L} \quad \text{where} \quad \bar{w}_i = \left(\sum_{s'} T_{s'}(w_{i,s'})^\vartheta \right)^{1/\vartheta}$$

FURTHER READING

- ▶ Caliendo and Parro 2018 (RESTUD): Classic paper introducing input-output linkages into quantitative model of trade a la Eaton Kortum (2002) without labor mobility
- ▶ Lee 2020 (JIE): nice paper adding occupations to IO analysis
- ▶ Caliendo Parro Dvorkin 2019 (ECMA): Generalize the Caliendo Parro setup to a world with countries, regions within, and forward looking migration
- ▶ Eckert 2019 (JMP): Multisector Armington model with Input-output structure across sectors, occupations, skill groups and migration across regions

**NON-HOMOTHETIC
DEMAND**

OVERVIEW

- ▶ Moving from CES (or Cobb Douglas) to non-homothetic preferences is appealing for some questions
 - ▶ Most salient: richer people spend a smaller fraction of their income on housing
- ▶ Several papers introduce non-homothetic preference into the spatial model:
 - ▶ Eckert and Peters (2018): "PIGL" preferences from Boppart (ECMA 14)
 - ▶ Finlay and Williams (2021): Non-homothetic CES preferences