# **INTERNATIONAL TRADE - ECON 245** FABIAN ECKERT











### SETUP

- Armington trade model with migration
- Contribution 1: Model is dynamic, so object are indexed by time period t
  - Agents live for two periods: childhood and adulthood
    - Overlapping generations-like model but without altruism
    - Kills dynamics for individual decisions, but allows to think about time
- Contribution 2: productivities and amenities depend not only on current but also past local populations; potential for different paths of the economy!

# MIGRATION

- $\triangleright$  Consider a distribution of workers across locations  $L_{rt}$  at time t
- Each of these workers has an "invisible" child during time period t
- > At the end of time t the parents die and the children choose a location to work
- ▶ Migration costs  $\mu_{ij}$  to move from *i* to *j*, paid in utility terms, wlog  $\mu_{ii} = 1, \mu_{ij} \ge 1 \forall j \neq i$
- Each agent  $\omega$  has idiosyncratic preferences for each destination  $\eta_i^{\omega}$ 
  - Total utility in each destination is given

ve by: 
$$W_j^{\omega} = U_j \frac{W_j}{P_j} \eta_j^{\omega}$$

# MIGRATION

- Each agent draws the  $\eta_i^{\omega}$  i.i.d. from a Frechet distribution with scale parameter 1 ("mean") and shape parameter  $\theta$  ("dispersion")
- > An individual young adult in birthplace location *i* hence solves:  $j^{\star} = \arg \max\{W_i^{\omega}\}$
- But then we obtain an analytic expression for the fraction of young adults choosing destination *j*:

$$\frac{\left(\mu_{ij}U_{j}w_{j}\right)}{-}$$

 $(P_j)^{U} \left( \frac{\mu_{ij} U_j w_j}{P_j} \right)^{U}$  $\pi_{ijt} = \frac{\left(\mu_{ik}U_k w_k/P_k\right)^{\theta}}{\sum_k \left(\mu_{ik}U_k w_k/P_k\right)^{\theta}} \equiv \frac{\left(\mu_{ij}U_j w_j\right)^{\theta}}{\Lambda_i}$ 

# EQUILIBRIUM

- Labor/goods market clearing:
  - Vhere  $\lambda_{ij}$  are the standard Armington trade shares
- The spatial equilibrium condition is now simply

### The equilibrium system is somewhat simpler than in the free mobility Armington case

 $L_i w_i = \sum \lambda_{ij} L_j w_j$ 

 $L_{i,t} = \sum \pi_{k,i} L_{k,t-1}$ 

• Where the migration shares  $\pi_{k,i}$  summarize optimal migration choices of agents

### **OBSERVATIONS**

- and just stay where you were born
- The fraction of young adults moving is also the ex-ante probability of an individual worker moving before they learn their productivity
- by the fraction moving to k", i.e.,  $\pi_{ik}/\pi_{ik'}$  is independent of  $\pi_{ik''}$ . Realistic?
- Next two observations deserve their own slides...

Since  $\mu_{ii} = 1$  and  $\mu_{ij} \ge 1 \forall j \ne i$  there is an incentive to forego wage differences

> The fraction of workers moving to k relative to those moving to k' is unaffected



## **MIGRATION GRAVITY**

- We now get a true gravity equation for migration flows!
- The number of people moving from i to j in period t to work there can be written as:

- > Taking logs:  $\log(L_{ii,t}/L_{i,t-1}) = \theta \log(\mu_{ii}) + \theta \log(U_i w_i/P_i) + \log(1/\Lambda_i)$
- > Parameterising migration costs as a function of distance:  $\mu_{ii} = d_{ii}^{\kappa}$

 $\log(\pi_{ii,t}) = \theta \kappa \log(d_{ii}) + \varphi_i + \xi_i + \epsilon_{ii}$ 

 $L_{ij,t} = \frac{\left(\mu_{ij}U_j w_j / P_j\right)^{\theta}}{\Lambda_i} L_{i,t-1}$ 

# MIGRATION GRAVITY AND THE DATA

- This is usually estimated using a Poisson Maxi 2006, paper+stata package!)
  - Rewrite the equation:  $\pi_{ij,t} = \exp(\theta \kappa \log(d_{ij}) + \varphi_j + \xi_i)\tilde{\epsilon}_{ij}$ 
    - This looks a lot like Poisson probability mass function!
    - Easy to construct likelihood of observed migration behavior
- Running these regression in the data:
  - The US census provides state of birth and current state for workers for the last 200 years can easily construct these lifetime migration probabilities!
  - Linked historical census data another great way to run these regressions

### > This is usually estimated using a Poisson Maximum Likelihood estimator (see Silva and Tenreyro

### **GROSS VERSUS NET MIGRATION FLOWS**

- An essential distinction in models of migration is net versus gross flows
- Gross flows between locations i and j are given by:

$$L_{ij,t} = \pi_{ijt} L_{i,t-1}$$

- So there are flows in both directions!
- Net flows are then given by:

$$\Delta = L_{ij,t} - L_{ji,t}$$

Net flows are only positive for one of the two locations

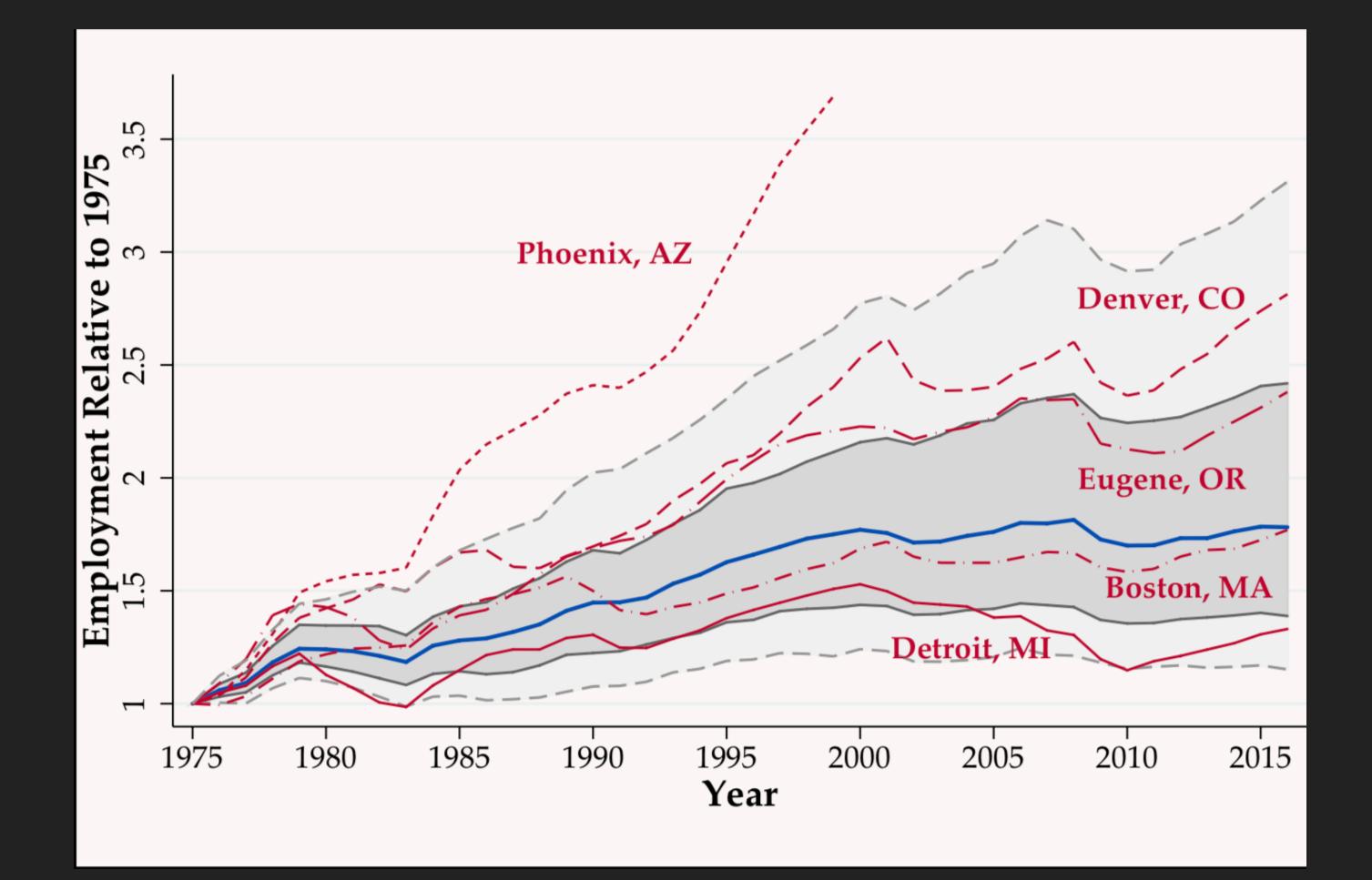
and  $L_{ji,t} = \pi_{jit}L_{j,t-1}$ 

 $= \pi_{ijt} L_{i,t-1} - \pi_{jit} L_{j,t-1}$ 

### **GROSS VERSUS NET MIGRATION FLOWS**

- Suppose we are in a long run steady state of the model
  - Then net flows are zero between regions but there are still gross flows.
- Now suppose we raise  $A_i$  for some location i
  - This sets into motion several periods of adjustment in which net flows between regions are non-zero
    - In particular there will be positive inflows into region i from the other regions

### **GRAPH FROM WALSH (2020): NET POPULATION GAINS ACROSS US CITIES**



### ADDITIONAL BELLS AND WHISTLES: SLOW DOWN ADJUSTMENT DYNAMICS

- We can add a "fixed cost of moving": suppose each period only a fraction  $\lambda \in (0,1)$  make a moving decisions
  - Then the total outflow of region *i* is given by:

$$L_{i,t} = \sum_{k} \pi_{k,k}$$

looking agents!

 $\lambda L_k + (1 - \lambda)L_{i,t-1}$ 

This way can get arbitrarily slow adjustment dynamics even without forward



# PATH DEPENDENCE IN ALLEN AND DONALDSON

- about path dependence
- > AA introduce path dependence in amenities and productivities as follows:

$$A_{it} = \bar{A}_{it} L^{\alpha_1}_{it} L^{\alpha_2}_{it-1}$$

- entail different distributions of productivities and amenities today!

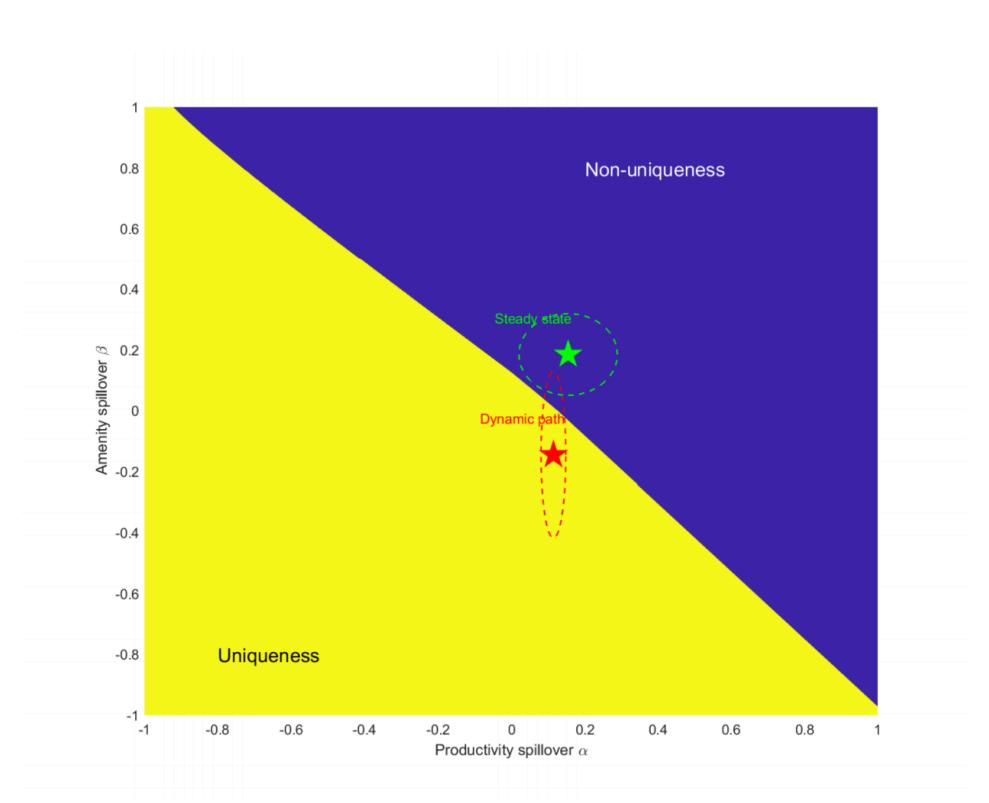
One you have set up this overlapping generations structure, can think seriously

and 
$$u_{it} = \bar{u}_{it} L_{it}^{\beta_1} L_{it-1}^{\beta_2}$$

Suddenly initial conditions matter: two different worker distributions at time t-1

Given initial conditions get unique dynamic path, but steady state differs by initial condition so there are multiple steady states: history can now matter!

## PATH DEPENDENCE IN ALLEN AND DONALDSON

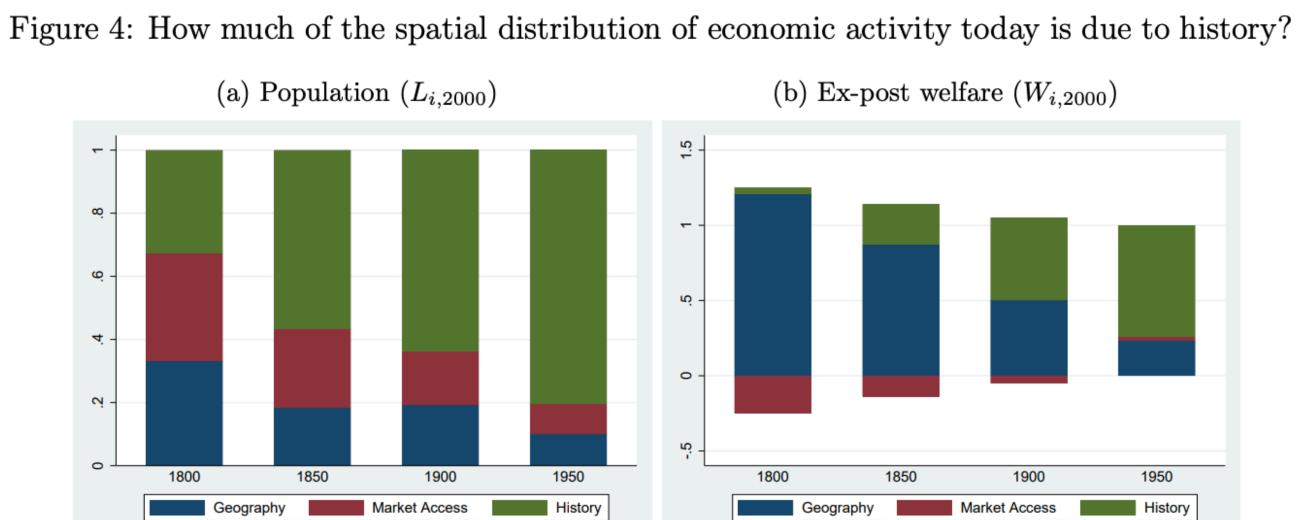


2. Confidence intervals are shown with dashed lines.

### Figure 3: Agglomeration spillover parameter estimates

*Notes*: This figure illustrates the parameter estimates (holding  $\sigma$  and  $\theta$  constant) obtained in Section 3.3. The red star denotes  $\widehat{\alpha}_1$  and  $\widehat{\beta}_1$ , which lies in the yellow region of equilibrium uniqueness following Proposition 1. The green star denotes  $\widehat{\alpha}_1 + \widehat{\alpha}_2$  and  $\widehat{\beta}_1 + \widehat{\beta}_2$ , which lies in the blue region, indicating the possibility of multiple steady-states following Proposition

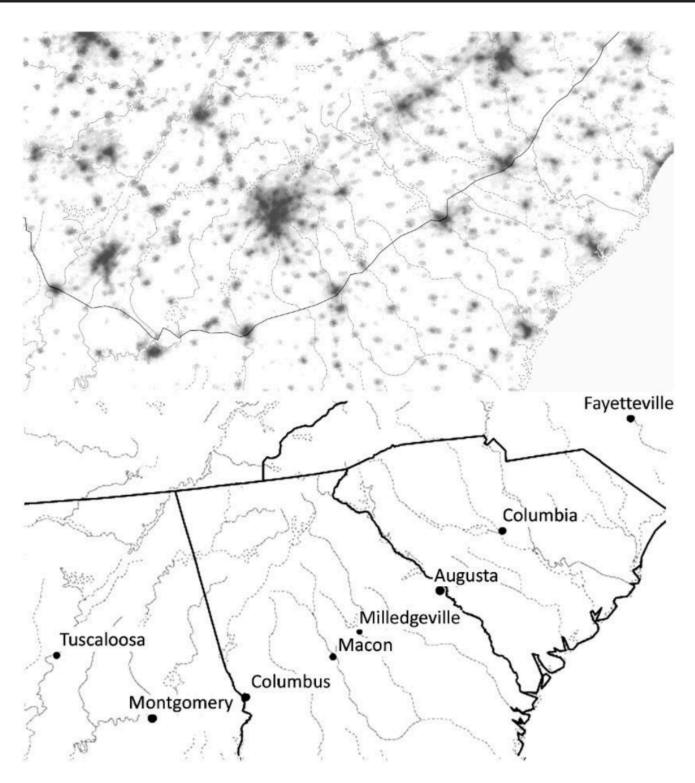
### PATH DEPENDENCE IN ALLEN AND DONALDSON



equivalent for ex-post welfare  $(W_{i,2000})$ .

*Notes*: This figure presents the variance decomposition of the observed spatial distribution of economic activity in the year 2000 into three components, as per equation (33): geography fundamentals (i.e. the complete history of realizations of productivities  $\overline{A}_{it}$  and amenities  $\bar{u}_{it}$  from t = 0 until the present), market access (i.e. the complete history of goods market access  $P_{it}$  and labor market access  $\Lambda_{it}$  from t = 0 until the present), and history (i.e. the population distribution in  $t = 0, L_{i0}$ ). The decompositions shown correspond to four choices of initial year t = 0: 1800, 1850, 1900, and 1950. Panel (a) presents the decomposition for the observed distribution of population in the year 2000  $(L_{i,2000})$ , and panel (b) presents the

### **RELATED: PORTAGE AND PATH DEPENDENCE BY BLEAKLEY AND LIN**



### FIGURE II Fall-Line Cities from Alabama to North Carolina

The map in the upper panel shows the contemporary distribution of economic activity across the southeastern United States, measured by the 2003 nighttime lights layer from NationalAtlas.gov. The nighttime lights are used to present a nearly continuous measure of present-day economic activity at a high spatial frequency. The fall line (solid) is digitized from *Physical Divisions of the United States*, produced by the U.S. Geological Survey. Major rivers (dashed gray) are from NationalAtlas.gov, based on data produced by the United States Geological Survey. Contemporary fall-line cities are labeled in the lower panel.

# CALIENDO, DVORKIN,

## INTRODUCTION

- lived agents.
- The difficulty: agents have to predict the path of wages and rents in each locations to make their moving decisions today
  - E.g.: Could be optimal to move to Denver today, because wages are high AND its close to Salt Lake City which will do well in three decades from now
    - There is now an option value to being in each location!

### Now we make the migration decision forward-looking, i.e., we have infinitely



### SETUP

- Armington trade model with migration; perfect competition; no forward looking decisions by firms
- Novelty is on the worker side:

  - $\triangleright$  Discount the future with  $\beta$
  - Redraw new preference shocks for each location each period
- Idiosyncratic amenity shocks are Gumbel distributed

### Workers live forever, "dynasties" (like Allen Donaldson but with altruism)

# MIGRATION DECISION

- The utility of worker  $\omega$  in location *i* today then looks as follows:  $v_{i,t}^{\omega} = W_i + \max_i \left(\beta E_{\eta_{t+1}}[v_{j,t+1}^{\omega}] - \frac{1}{2}\right)$

$$V_{i,t} = W_i + E_{\eta_t} [\max_{j} \left(\beta V_{j,t+1} - \mu_{ij} + \eta_{j}\right)]$$

into account all possible future draws of idiosyncratic shocks

$$-\mu_{ij} + \theta \eta_{jt}^{\omega}$$
) where  $W_i = u_i w_i / P_i$ 

Now define V<sub>i.t</sub> as the expected lifetime value of an agent in i before learning his idiosyncratic preference shock:  $V_{i,t} = E_{\eta_t}[v_{i,t}^{\omega}]$   $V_{i,t} = W_i + E_{\eta_t}[\max_j \left(\beta V_{j,t+1} - \mu_{ij} + \eta_{jt}^{\omega}\right)] = W_i + \theta \log \left[\sum_j \exp\left(\beta V_{j,t+1} - \mu_{ij}\right)^{\frac{1}{\theta}}\right]$ 

So V<sub>j,t+1</sub> summarizes the option value of being in location j tomorrow taking

# **MIGRATION DECISION**

involves the continuation value:

So now workers assess destinations not just for their current wage, but for the path of future wages and the strategic migration position they offer!

- "Option value" of each location matters

# The fraction of workers moving from i to j is now like the Gumbel one we discussed, BUT ue: $m_{ij} = \frac{\exp\left(\beta V_{j,t+1} - \mu_{ij}\right)^{\frac{1}{\theta}}}{\sum_{j} \exp\left(\beta V_{j,t+1} - \mu_{ij}\right)^{\frac{1}{\theta}}}$

> Option value does not depend on idiosyncratic shocks since they a redrawn each period



## **STEADY STATE EQUILIBRIUM**

Labor/goods market clearing: The spatial equilibrium condition is now simply  $L_{j,t} = \sum_{k} I$ In steady stage, by definition  $V_j = V_{j_j}$  $V_i = W_i + \theta \log$ 

 $L_i w_i = \sum_j \lambda_{ij} L_j w_j$ 

$$\int_{a}^{\pi} \pi_{k,j} L_{k,t-1}$$

$$\int_{a}^{t} = V_{j,t-1} \text{ so can solve for the option value} \sum_{k} \exp\left(\beta V_{k} - \mu_{jk}\right)^{\frac{1}{\theta}}$$

# **SOLVING FOR THE TRANSITION**

- Suppose now some fundamental changes  $\Theta = \{A_{it}, u_{it}, \tau_{iit}, \mu_{iit}\}$ 
  - Could be one-off changes or and entire path of changes
- In theory can then solve in two steps:
  - Step 1: Compute new steady state where  $V_i = V_{i,T} = V_{i,T-1}$  for some T using previous slide
  - Step 2: Guess path of value functions  $\{V_{it}\}_{t=0}^T$  and then simulate forward from initial conditions in period 0, iterate





# SOLVING FOR THE TRANSITION: DYNAMIC HAT ALGEBRA

- $\blacktriangleright$  Solving the equilibrium of the model requires knowing  $\Theta$  at each point in time
- > As we added countries, regions, sectors, the number of parameters grows fast
- $\blacktriangleright$  Rewrite the model in changes: given data for the initial period, can solve for entire transition to new steady state without knowing  $\Theta$



### **SOLVING FOR THE TRANSITION: DYNAMIC HAT ALGEBRA**

The model in changes:

$$\exp(V_{it+1} - V_{it}) = (\hat{w}_{it+1} / \hat{P}_{it+1}) \left[ \sum_{j} \exp\left(V_{jt+2} - V_{jt+1}\right)^{\beta/\theta} \pi_{ijt} \right]$$
  
shares:  
$$\pi_{ijt+1} = \frac{\pi_{ijt} \exp(V_{jt+2} - V_{jt+1})^{\beta/\theta}}{\sum_{k} \pi_{ikt} \exp(V_{kt+2} - V_{kt+1})^{\beta/\theta}}$$

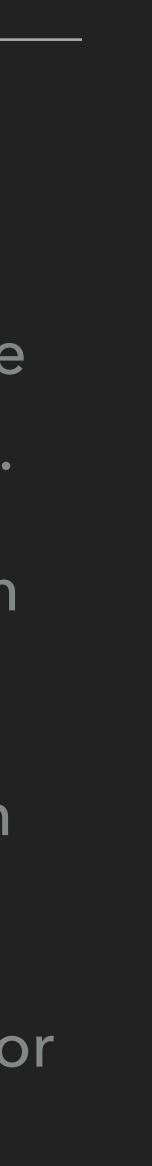
Migration

- And migration itself:
  - where as always  $\hat{x}_{t+1} = x_{t+1}/x_t$

 $L_{i,t} = \sum \pi_{k,i} L_{k,t-1}$ k

## ALGORITHM IN BROAD SKETCHES

- Guess path for  $\{V_{it+1} V_{it}\}_{t=0}^{T}$ , *T* is also a guess. Note that we have data at time t = 0 and that  $\{\hat{\Theta}_t\}_{t=0}^{T}$  which is the "shock" the response to which we compute.
- Use this to solve for population in each region in each location at each point in time using the migration shares in changes
- Use these populations to solve static labor marker equilibrium at each point in time.
- Use new steady state in T and path of wages and prices to infer a new guess for  $\{V_{it+1} V_{it}\}_{t=0}^{T}$







# INTRODUCTION

- Why we discuss the paper:

  - idiosyncratic preferences
  - It introduces correlated Frechet shocks into the migration literature
  - It highlights the limits of the Frechet approach in modeling selection

It produces a gravity equation for migration, while still being a static model It highlights selection on productivity, so far we have only seen selection on

### **EMPIRICAL OBSERVATIONS**

- origin workers who choose to migrate.
- distance earn more on average.
- route their average wage is lower.
- Fact 4: Wages are higher for longer distance migrants due to selection.
- wages for migrants

Fact 1: The further the distance between origin and destination, the lower the share of

Fact 2: Controlling for origin and destination fixed effects, workers the migrated further

Fact 3: Controlling for origin and destination fixed effects, if more people migrate on a

Fact 5: Controlling for origin fixed effects, high destination amenities, yields lower average





### SETUP

- Economy consists of N locations
- A mass of workers  $L_o$  is born into each location o
- - locations depending on their skills and local industrial structure
- $\triangleright$  Workers then choose their labor market of employment indexed by d

Standard Armington setup: region-specific varieties, CES, perfect competition

Workers receive idiosyncratic productivity shocks for each possible destination

Reduced form way of modeling that some workers are productive in some

### **LOCATION CHOICES**

- The total utility in destination d of someone born into o is:

> Worker *i* from origin *o* could supply the following human capital in destination *d*:  $h_{od}^i = s_d^i q_o$ 

 $\triangleright$  Where  $q_o$  can be thought of as a measure of the quality of education in origin

 $U_{od}^{i} = u_{d}\mu_{od}w_{d}h_{od}^{i}$ 

▶  $u_d$ : amenities,  $w_d$ : wages per unit of HC,  $\mu_{od}$ : moving cost,  $\mu_{oo} = 1, \mu_{od} < 1 \forall o \neq d$ 



## **LOCATION CHOICES**

- The idiosyncratic productivity shocks are
   F(s<sub>1</sub>, ..., s<sub>N</sub>) = exp{- [ \sum\_d \su
  - The entries of the vector of shocks for an agent are no longer i.i.d!
  - >  $1/\theta$  measures dispersion of shocks: importance of comparative advantage
  - $\blacktriangleright\ \rho$  measures the correlation between shocks of a given worker
    - $\blacktriangleright \ \rho \rightarrow 1$  talent becomes unidimensional;  $\rho \rightarrow 0$  back to i.i.d shocks!

re drawn from a *multivariate* Frechet  

$$\begin{bmatrix} \frac{\theta}{1-\rho} \\ s_d^{\frac{1-\rho}{1-\rho}} \end{bmatrix} \equiv \exp\{-\left[\sum_{d} s_d^{\theta}\right]^{1-\rho}\}$$

### LOCATION CHOICES

- The resulting location choices have familiar expressions:
- flows:

# $\pi_{od} = \frac{(u_d \mu_{od} w_d q_o)^{\theta}}{\sum_d (u_d \mu_{od} w_d q_o)^{\theta}} = \frac{(u_d \mu_{od} w_d)^{\theta}}{\sum_d (u_d \mu_{od} w_d q_o)^{\theta}}$

Importantly, origin effects that are not indexed by destination  $(q_{o})$  cancel out!

So different from the models thus far we get a gravity equation for migration

 $L_{od} = \pi_{od} L_o \Rightarrow \log(L_{od}) = \delta^o + \delta^d + \theta \log(\mu_{od})$ 

Can run this exact regression in US Census data from IPUMS (across states)

# SELECTION

- The Fréchet assumptions leads to two more nice analytical expressions:  $E(s_d \mid d^{\star} = d) = \pi_{od}^{-\frac{1}{\theta}} \overline{\Gamma} \quad \text{and} \quad \overline{w}_{od} = w_d q_o \pi_{od}^{-\frac{1}{\theta}} \overline{\Gamma}$ 
  - Selection: the fewer people move the higher their average productivity
- Only workers with very specific talent make long distance moves
  - Model predicts that American expats should earn more on average than Americans at home
- Large wage gap across locations may then not imply high returns to moving people: moving people would lower their average productivity and wages

### SELECTION

One other implications of the model

$$\bar{w}_{od}$$

$$\bar{w}_{od'}$$

- Observations:
  - No difference in wages across destinations without frictions/amenities
  - Observed wage gaps then are due to frictions that induce selection
    - > Young (2013) spatial wage gaps are all about selection, this is not due here
  - on average, least productive workers.
    - sorting)

 $\frac{u_d}{u_{d'}} \frac{\mu_{od}}{\mu_{od'}}$ 

Strange implication: destination wage difference do not show up: inherently productive locations attract the,

Assignment models (see Gaubert 2018, or Bilal Rossi-Hansberg 2020) have more realistic notion of

