Spatial Growth

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Introduction

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Problem for a competitive/neoclassical theory of growth!

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- As you might imagine, a lot depends on the shape of $g()$

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- Upshot: more varieties act like greater TFP A_t

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… but change *g* and everything changes

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- Different properties!

Growth depends on market size *growth*

Growth rate correct, level of income not

Growth rate *invariant* to policy

Idea Diffusion

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TABLE III
GEOGRAPHIC MATCHING FRACTIONS

	1975 Originating cohort			1980 Originating cohort		
	University	Top corporate	Other corporate	University	Top corporate	Other corporate
Number of citations	1759	1235	1050	2046	1614	1210
Matching by country						
Overall citation matching percentage	68.3	68.7	71.7	71.4	74.6	73.0
Citations excluding self-cites	66.5	62.9	69.5	69.3	68.9	70.4
Controls	62.8	63.1	66.3	58.5	60.0	59.6
t-statistic	2.28	−0.1	1.61	7.24	5.31	5.59
Matching by state						
Overall citation matching percentage	10.4	18.9	15.4	16.3	27.3	18.4
Citations excluding self-cites	6.0	6.8	10.7	10.5	13.6	11.3
Controls	2.9	6.8	6.4	4.1	7.0	5.2
t-statistic	4.55	0.09	3.50	7.90	6.28	5.51
Matching by SMSA						
Overall citation matching percentage	8.6	16.9	13.3	12.6	21.9	14.3
Citations excluding self-cites	4.3	4.5	8.7	6.9	8.8	7.0
Controls	1.0	1.3	1.2	1.1	3.6	2.3
t-statistic	6.43	4.80	8.24	9.57	6.28	5.52

Number of citations is less than in Table I because of missing geographic data for some patents. The *t*-statistic tests equality of the citation proportion excluding self-cites and the control proportion. See text for details.

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Other seminal empirical work:

- Ellison Glaeser 1997 JPE
- Arzaghi and Henderson 2006 Restud
- De La Roca and Puga 2018 Restud

Cross-Country Idea Diffusion

- At what level should we think about idea diffusion?
- Eaton Kortum 1999 IER: country level
- Output produced from

$$X_{nt}(j) = L_{nt}(j)^{1-\phi} K_{nt}(j)^{\phi}$$

$$ln(Y_t) = \int_0^1 ln\left(Z_{nt}(j) X_{nt}(j) \right) dj$$

where Z_{nt} is intermediate quality and X_{nt} is intermediate quantity

- Output freely traded (same price everywhere), intermediates non-traded

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$$Pr[\tau_{ni} < x] = 1 - e^{-\epsilon_{ni} x}$$

Diffusion friction

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In the limit, productivity grows with idea stock

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If γ=1, growth itself a function of frictions

- Your productivity μ_{nt} depends on the frictions between you and other productive, research intensive countries

Growth Decomposition

- Discipline idea flows with survey evidence on patents, calibrate other parameters to match relative productivities and research activity

TABLE 5
GROWTH DECOMPOSITION

Fraction of Productivity Growth in	Due to Research Performed in Germany	France	U.K.	Japan	U.S.
Germany	0.16 (0.02)	0.08 (0.01)	0.07 (0.01)	0.27 (0.02)	0.42 (0.04)
France	0.13 (0.01)	0.11 (0.02)	0.07 (0.01)	0.26 (0.02)	0.42 (0.04)
U.K.	0.15 (0.02)	0.07 (0.01)	0.13 (0.02)	0.32 (0.04)	0.33 (0.06)
Japan	0.14 (0.02)	0.07 (0.01)	0.07 (0.01)	0.35 (0.05)	0.36 (0.05)
U.S.	0.10 (0.01)	0.05 (0.02)	0.05 (0.01)	0.20 (0.03)	0.60 (0.06)

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- Within countries, people tend to move around

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Desmet, Nagy, Rossi-Hansberg 2018 JPE

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Ignore the moving costs, they don't matter

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- Technology shifter increases with local innovation, as well as diffusing from all other locations

- **Important**: Firms know that they will make zero profits in all periods, so innovation decision is not forward looking

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These two modules collapse to two integral equations a la. AA '14

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Dynamic

Upshot: Can solve for dynamic evolution

- Diffusion process too complicated to say much, has to be solved numerically
- Innovation stronger in higher density locations, but diffusion ensures a BGP eventually obtains
- In the long-run, populated places have the highest levels of technology (Africa and Asia!)
- Loosening migration frictions across countries lets people take better advantage of favorable amenities and trade geography (productivity follows people)

Rich set of counterfactuals and exercises

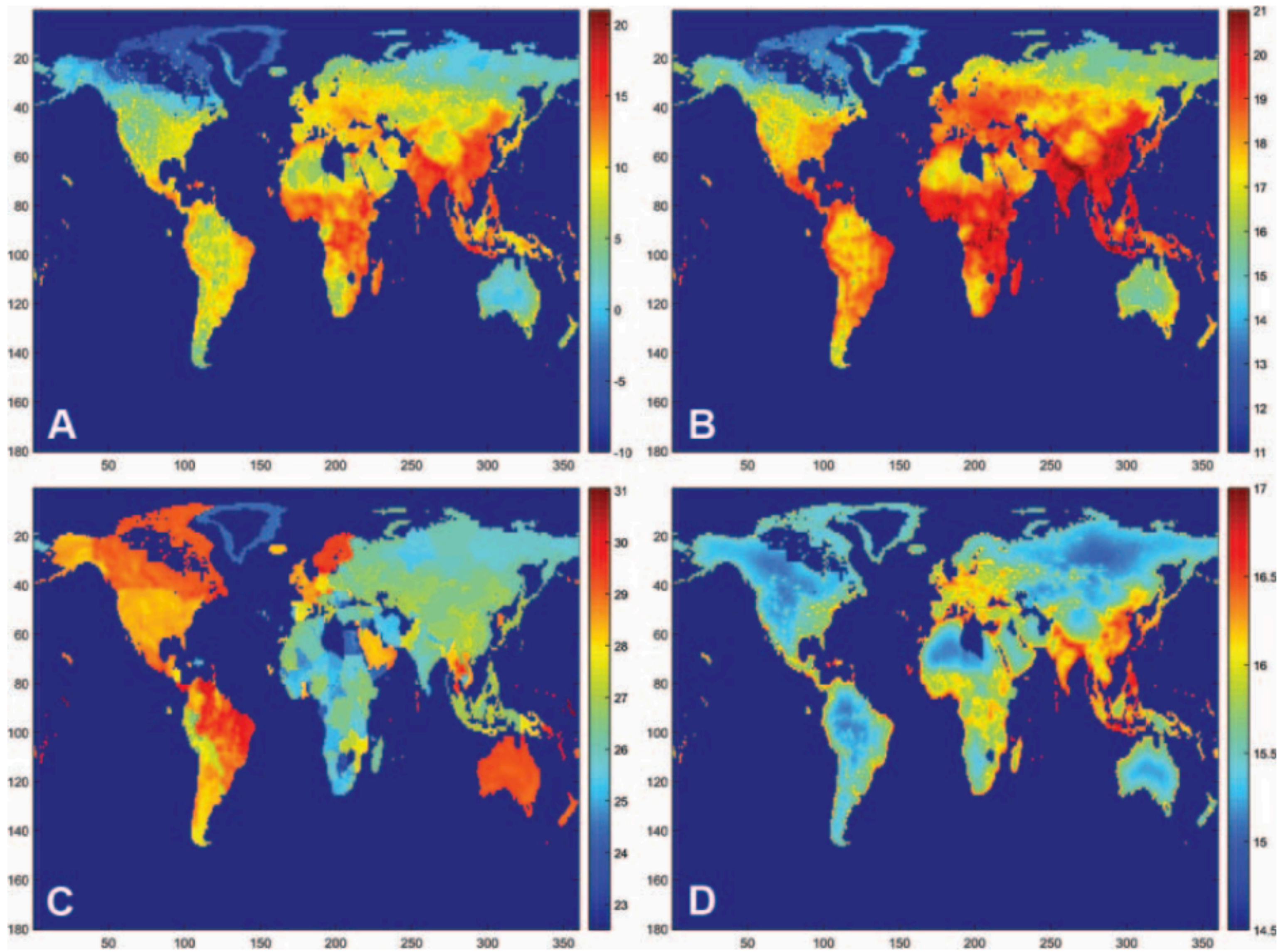

FIG. 3.—Equilibrium keeping migratory restrictions unchanged (period 600). *A*, Population density. *B*, Productivity: $[\tau_t(r)\bar{L}_t(r)^{\alpha}]^{1/\theta}$. *C*, Utility: $u_t(r)$. *D*, Real income per capita: $y_t(r)$.

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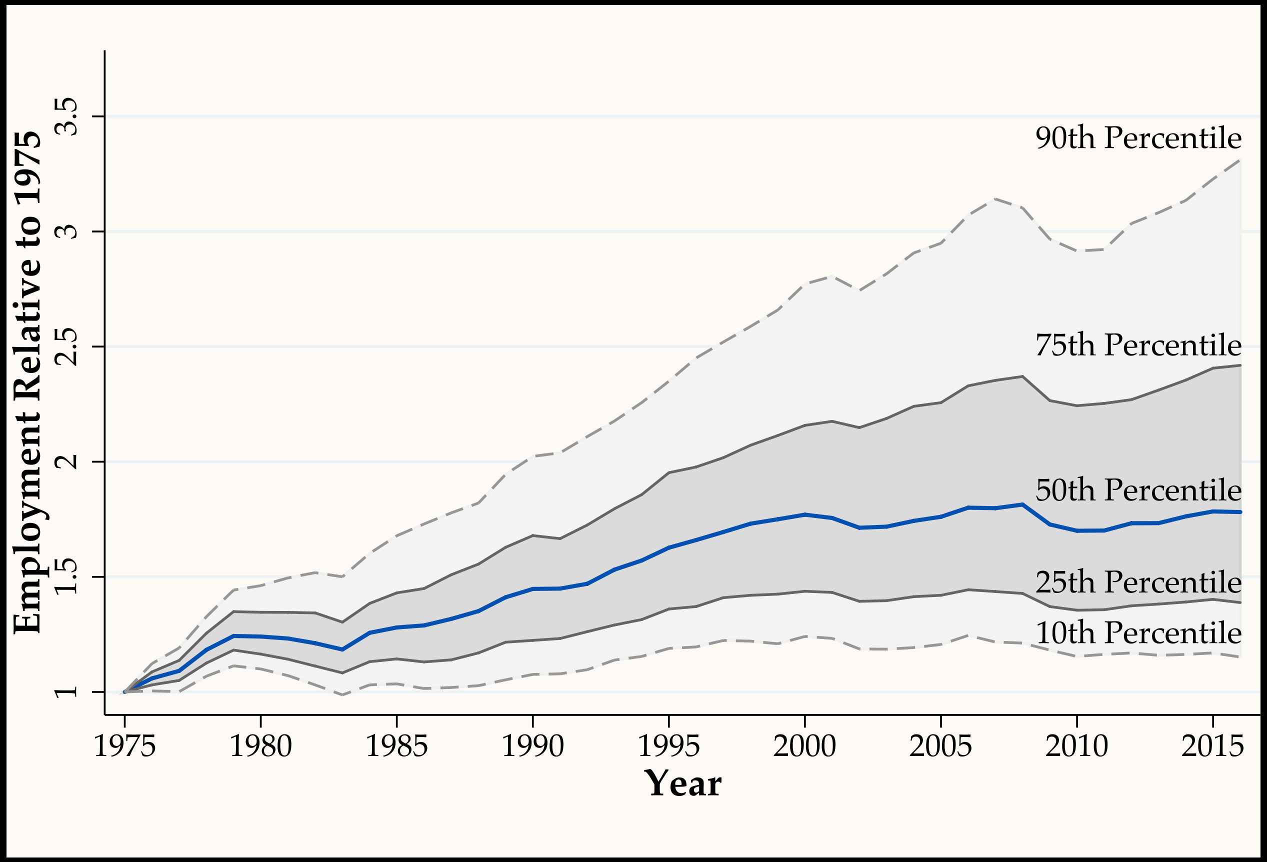

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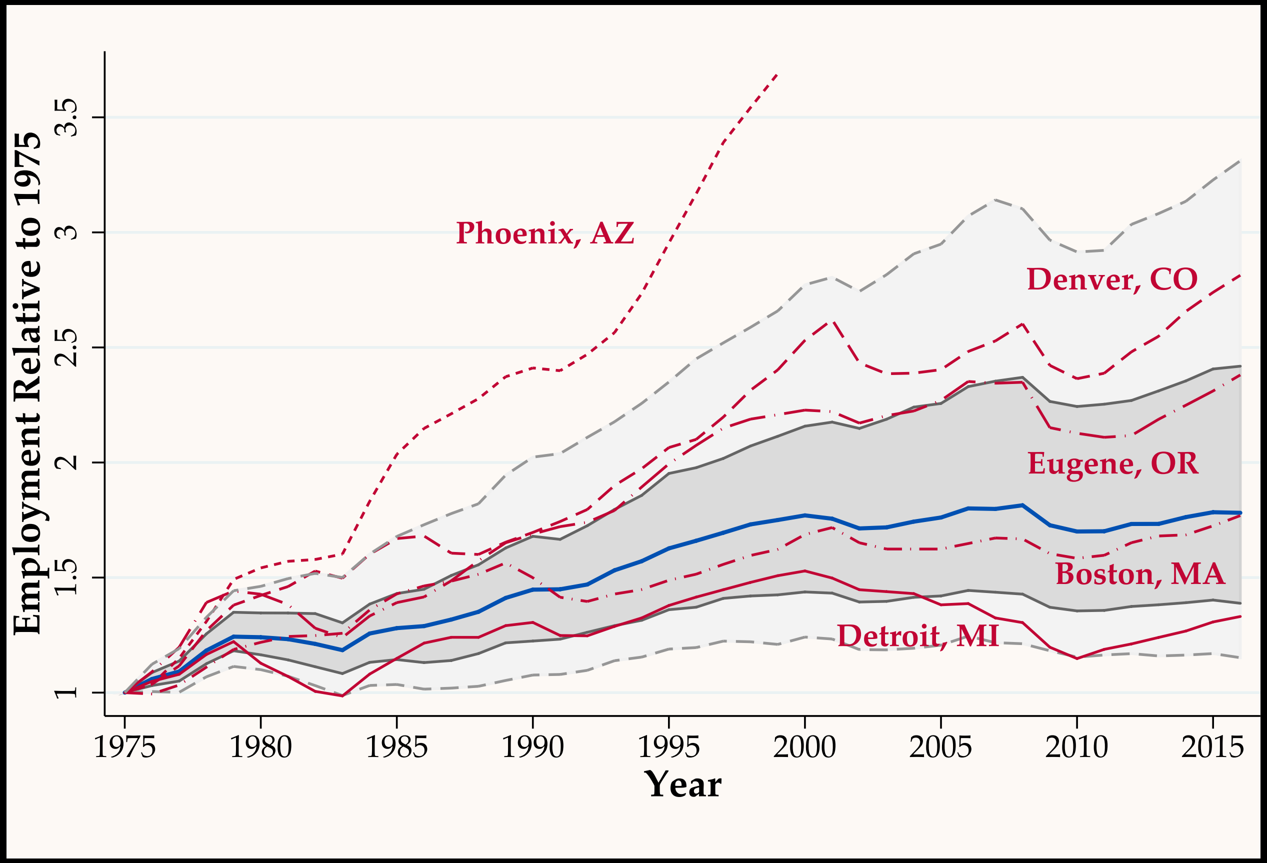

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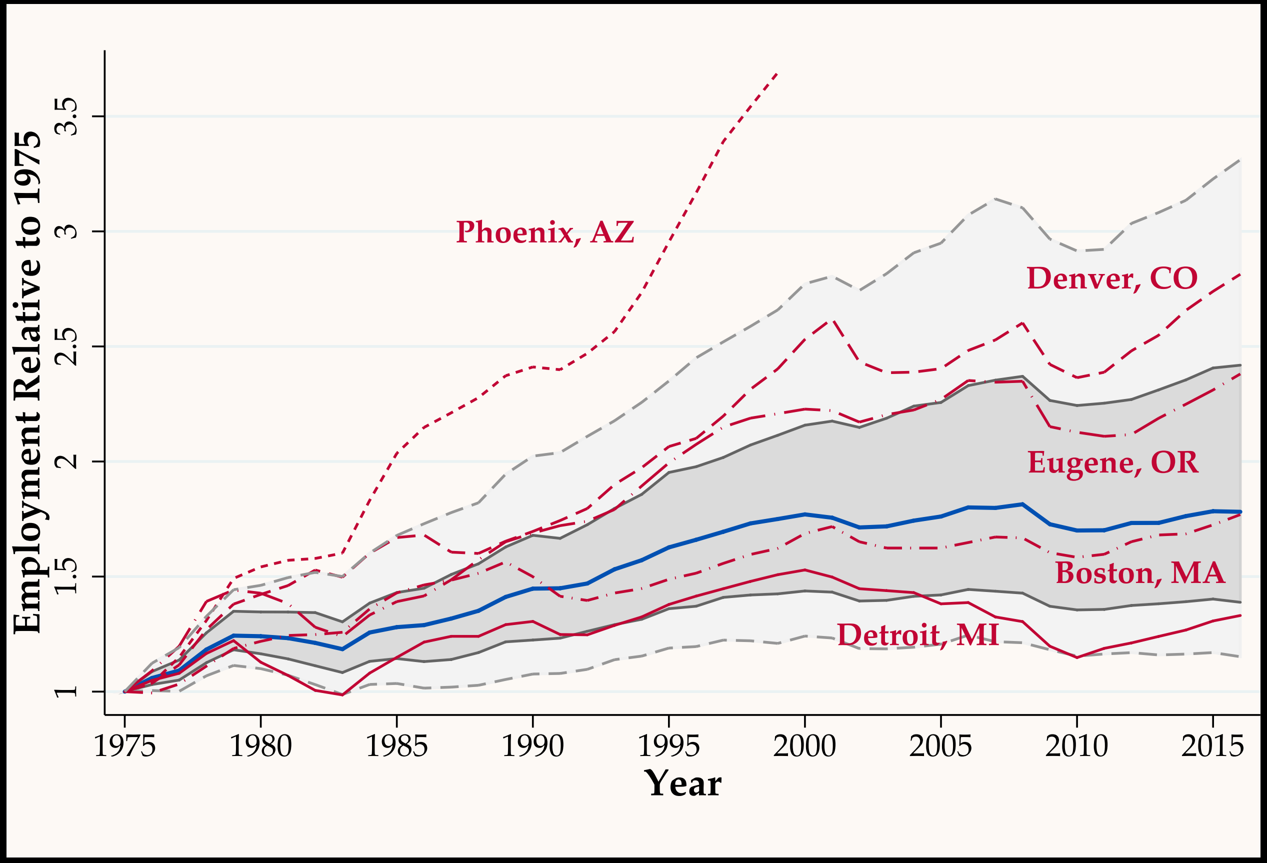
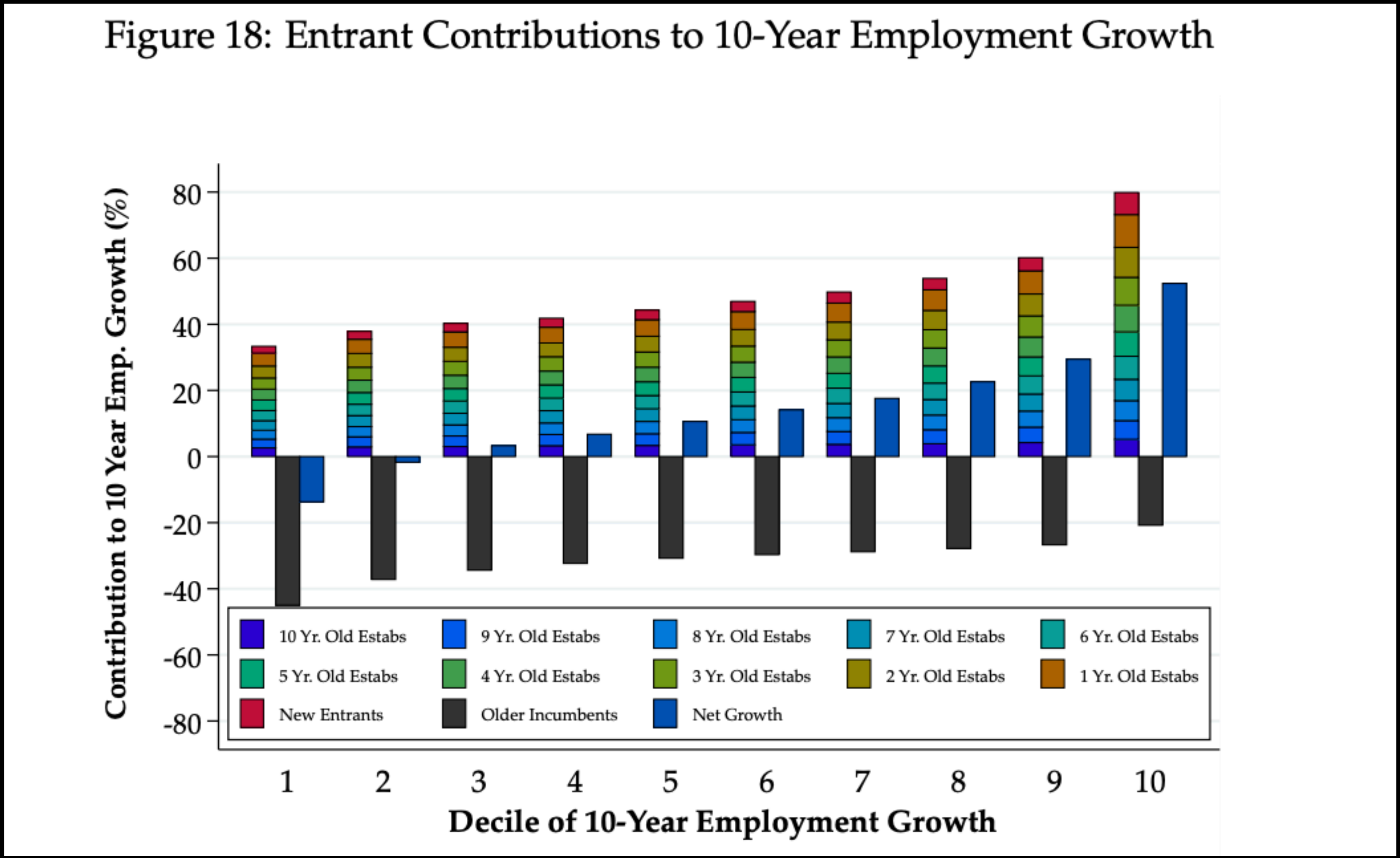


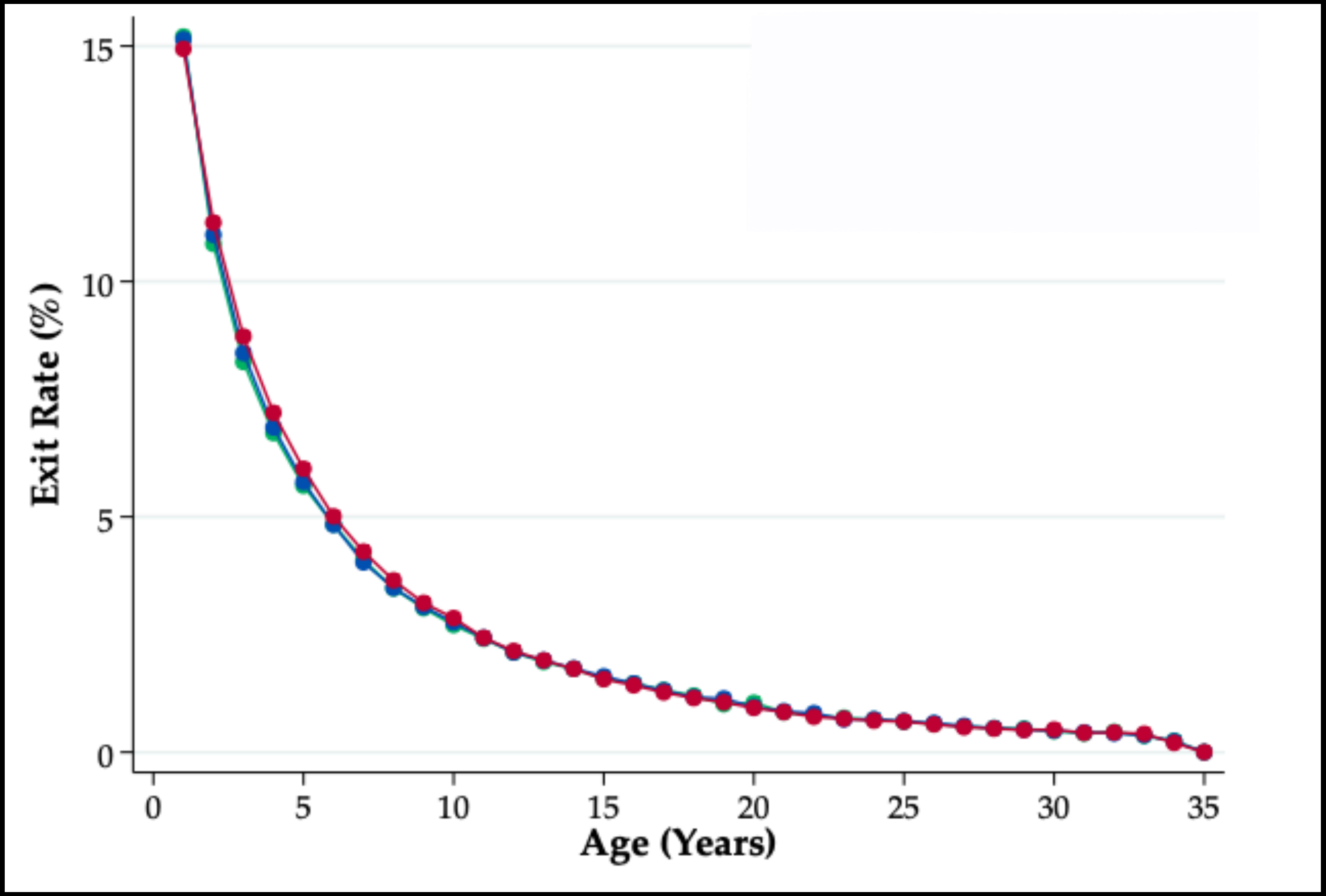
Walsh (2020)

Variation in entry accounts for most of these differences

Importance of Entry for Local Growth



Establishment lifecycle invariant across space



Location Choice and spatial inefficiency

- Upshot: dynamic location choice major component of city growth
- However, firms may not internalize the effects their startup decisions have on the local economy
- Result: *dynamic spatial misallocation*
- Booming cities grow too slow, dying cities persist too long
- Missed in static/SS models, long run steady state allocation *is* efficient. But fundamentals are constantly changing.

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Final good → C_t^i; Housing → H_t^i

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- Also directly raise demand for other firms

Core Idea (3)

Simplest case

$$\pi_{j,t} = \frac{1}{\sigma} N_{j,t}^{\frac{2-\sigma}{\sigma-1}} L_{j,t}$$

$$SR_{j,t} = \frac{1}{\sigma - 1} N_{j,t}^{\frac{2-\sigma}{\sigma-1}} L_{j,t}$$

- Competitive Euler equation:

$$\frac{\dot{C}_{j,t}}{C_{j,t}} = \frac{1}{\gamma} \left[\pi_{j,t} - \delta - \rho \right]$$

- No guarantee the rate of profit π_t is equal to the social return $SR_{j,t}$ with complementarities

- Two reasons: firms raise labor demand, induce more people to move to *j* (not present in aggregate models)

- Also directly raise demand for other firms

Main Paper

- Generally, growth of cities experiencing a shock to fundamentals is too slow.
- Midwest misery in part because South not creating jobs fast enough
- Embed this insight into a quantitative dynamic model with
    - Heterogenous firm dynamics
    - Land investment and housing capital
- Think about the mobility gains from optimal policy

Urban Macroeconomics

Urban Macroeconomics

- We've said less about my 2nd question: how does the macroeconomy affect the spatial distribution?

- Eckert Ganapati Walsh (2020): "*Skilled Scalable Services: The New Urban Bias in Economic Growth*"

- 2 big biases in recent growth:

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Skill Bias

Urban Bias

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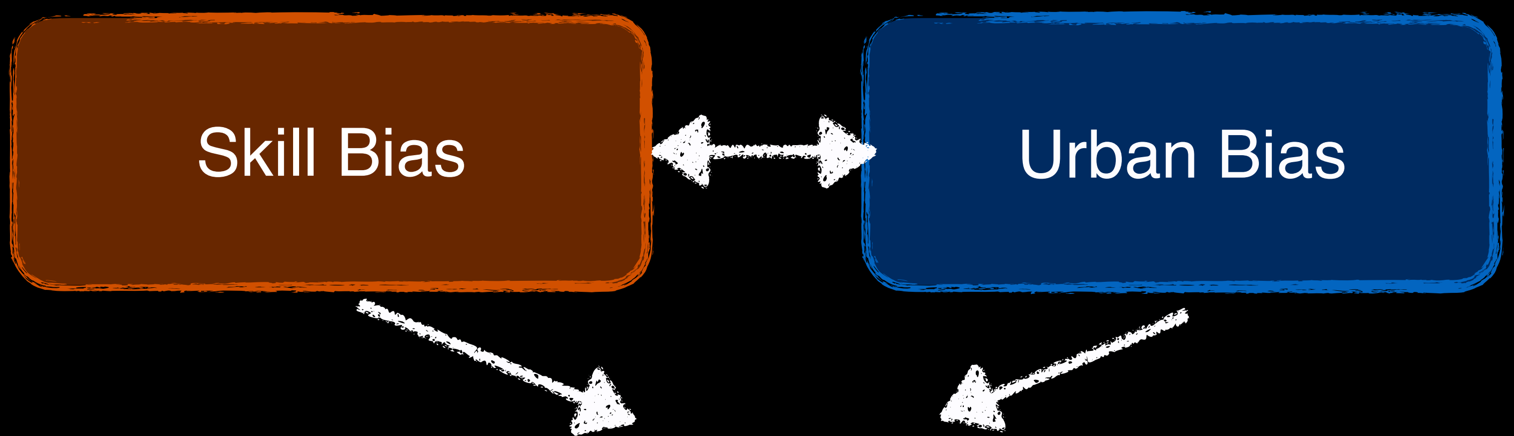
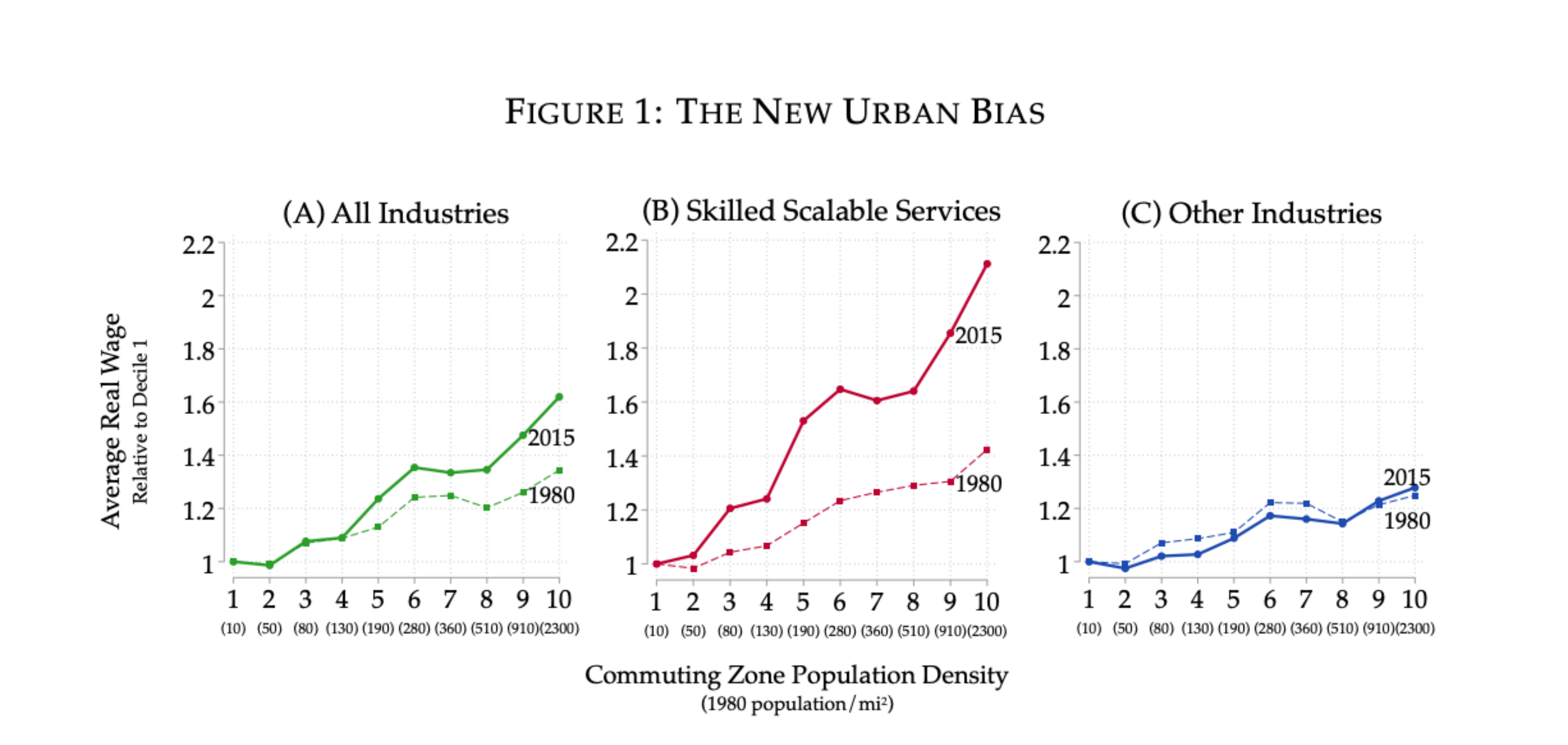


Are they driven by the same thing?

Urban Bias coming from Services



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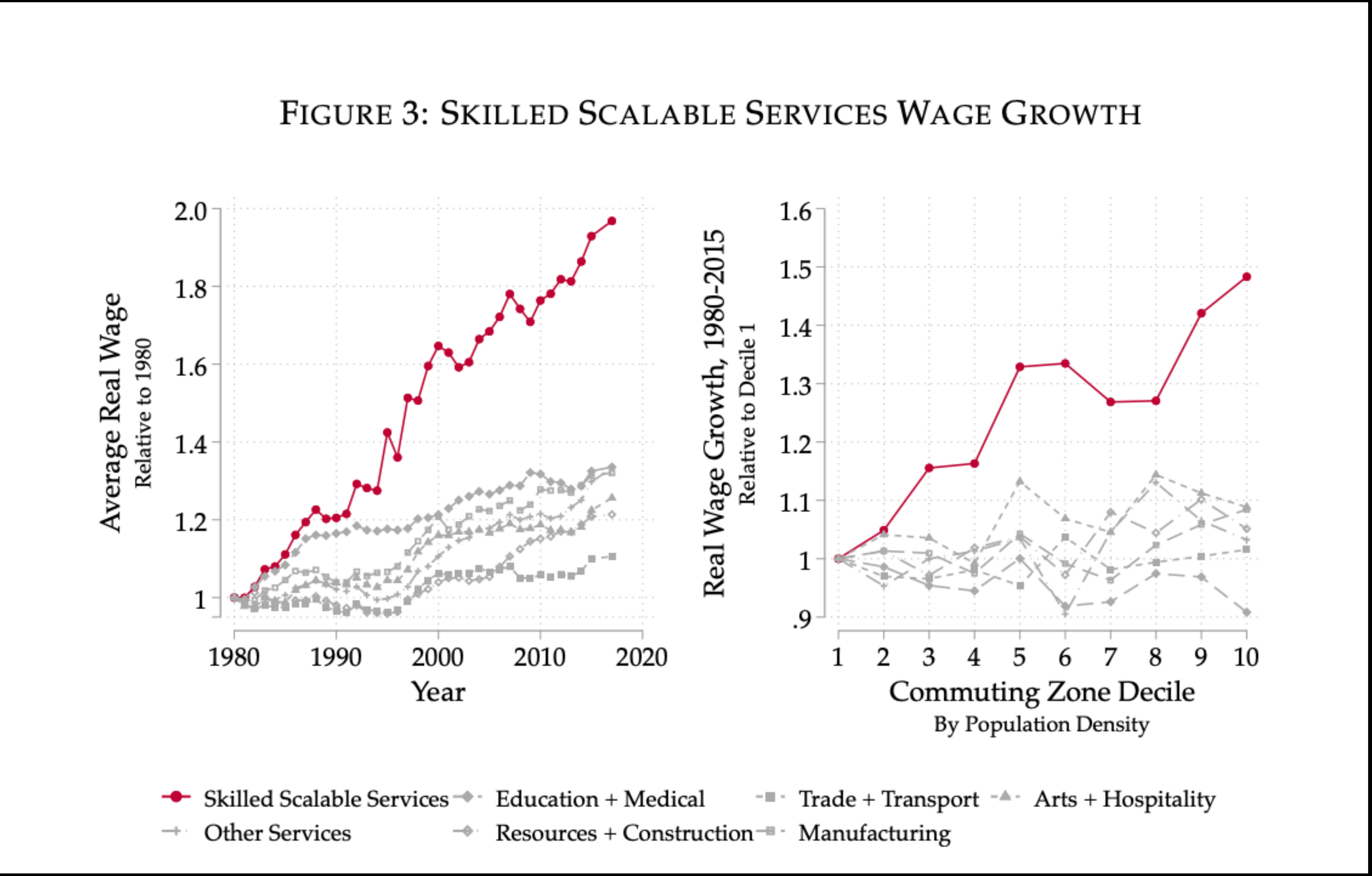

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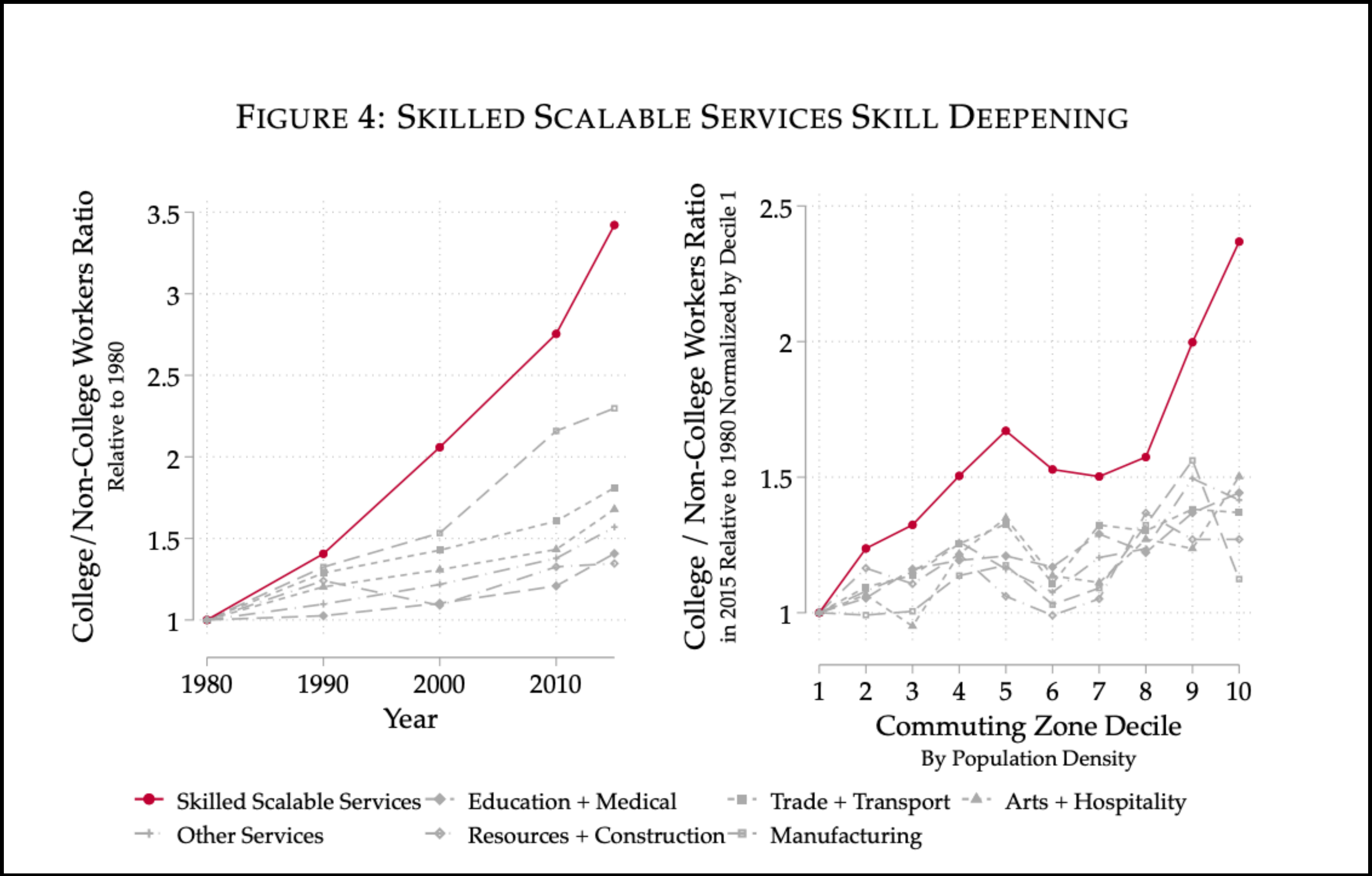

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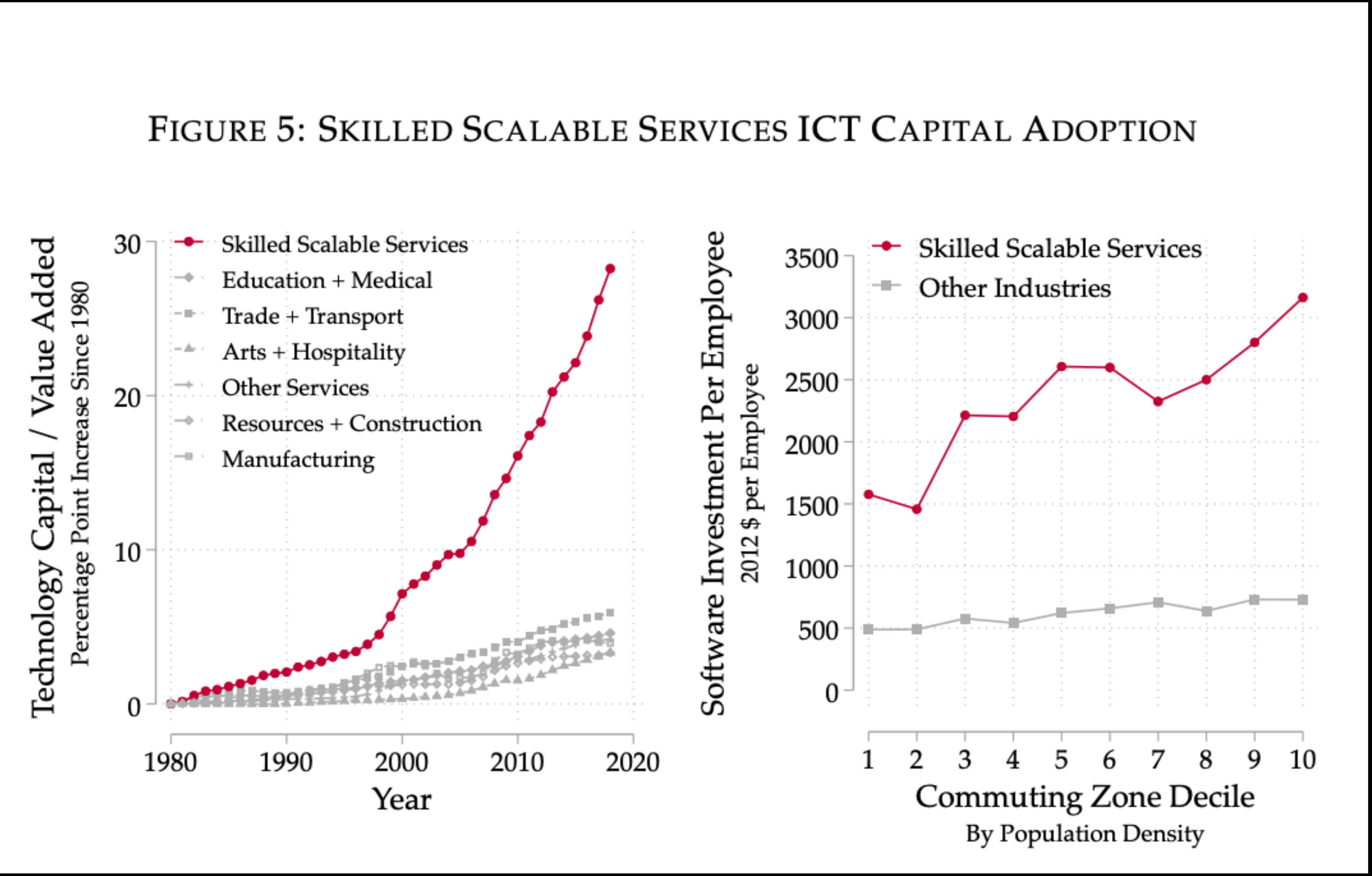
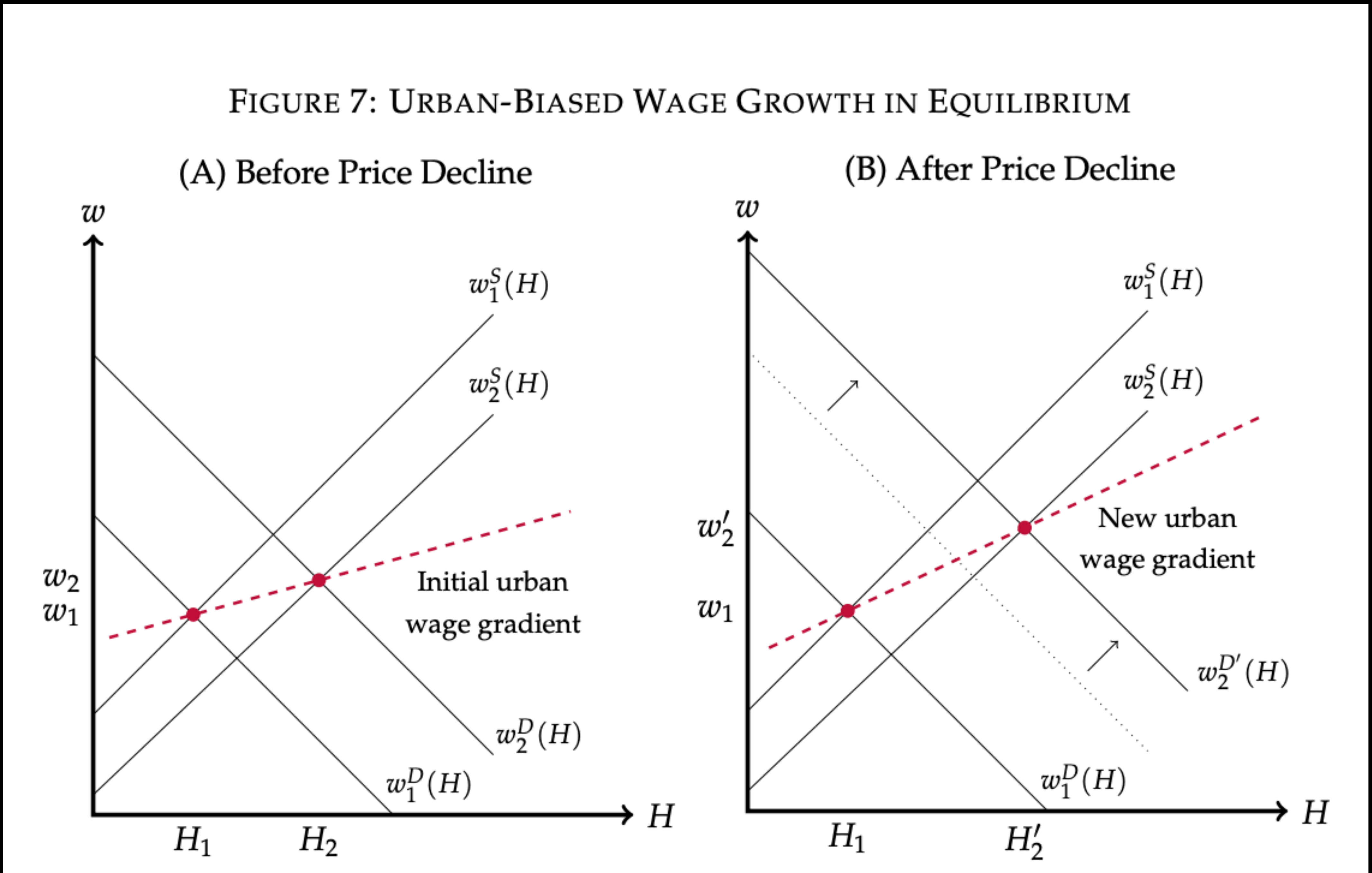


Our explanation

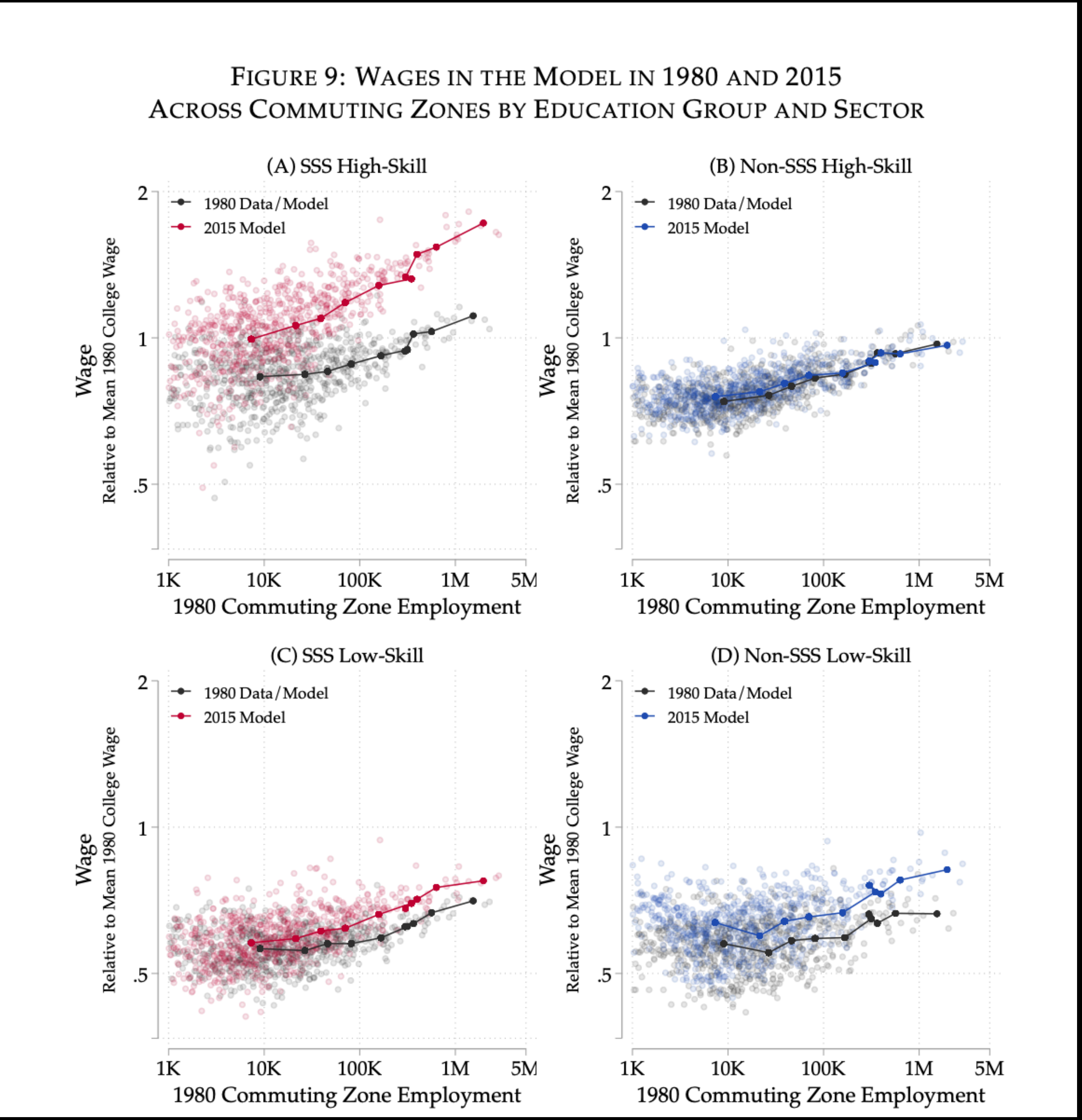
Our explanation

- Falling ICT prices at the macro level particularly important for SSS
- ICT allows them to increase their scale, and increases their use of skilled workers
- SSS firms have a comparative advantage in dense cities, so investment faster there
- Build model of heterogenous firms, fixed costs and non-homethetic production
- Trace spatial effects of aggregate price decline

Steepening Wage Gradient



Quantitative Wage Gradients



Other cool papers in this area:

- Arkolakis Peters Lee 2020 “European Immigrants and the Rise of the United States to the Technological Frontier"
- Eckert Peters 2019 “Spatial Structural Change”
- Duranton Puga 2019 “Urban growth and its aggregate implications”
- Davis, Fisher, Whited 2014, “Macroeconomic Implications of Agglomeration”
- Nagy 2020, “Hinterlands, city formation and growth: Evidence from the U.S. westward expansion”

Back to the future

- Spatial growth is about two questions:

1. How does the spatial distribution of economic activity affect the macroeconomy?

2. How does the macroeconomy affect the spatial distribution of economic activity?

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Can you help us get some answers?